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S. W. STRATTON, DIRECTOR

No. 100

DETERMINATION OF ABSOLUTE VISCOSITY BY SHORT-TUBE VISCOSIMETERS

BY

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CONTENTS

I. INTRODUCTION

In the determination of viscosity for technical purposes the Engler and the Saybolt Universal viscosimeters are the ones most commonly used in the United States. With the former it is customary to express results in "Engler degrees" obtained by dividing the time of discharge for the liquid being tested by the time for water at 20° C (68° F). With the latter instrument results are expressed as the time of discharge, in seconds, of the liquid being tested. There is, however, a growing demand for the expression of results in absolute or cgs units, but unfortunately the outlet tubes of these instruments are so short that there is considerable doubt as to the applicability of the formula showing the relation between absolute viscosity and time of discharge, which may be used in determining absolute viscosity by means of a long capillary tube.

Before this formula can be used it is necessary to decide upon the coefficient of the kinetic-energy correction and the correction to the measured length of tube, which is necessary to allow for other end effects. A formula must be selected for finding the average head and means found for determining under what conditions the formula is applicable; that is, within what limits of velocity there will be viscous flow as distinct from hydraulic or turbulent flow.

These subjects have been investigated both in the literature and by original experiments and the sources of error in viscosity determinations are considered at length. Tentative formulas are given for the determination of absolute viscosity from the time of flow with normal volumes put in and discharged, while diagrams are given which may be used to estimate the viscosity when abnormal volumes are used.

A method is given by which each operator of a viscosimeter may obtain the relation between viscosity and time of discharge for his own instrument, without use of the tube dimensions, which are difficult to determine with sufficient accuracy; and tables of viscosities of the necessary calibrating liquids are given in an appendix. This method is especially necessary because the Saybolt Universal viscosimeter has never been standardized as to the dimensions of its parts and their allowable tolerances, so that all data in regard to it must be regarded as merely approximate.

II. UNITS FOR EXPRESSING VISCOSITY

The words viscous and viscosity are not commonly used by every one who has observed that a viscous liquid will flow sluggishly. In automobile parlance a viscous oil is said to be "heavy," while an oil of lower viscosity is called "light," these terms having no reference either to the density, which is usually expressed in degrees Baumé, nor to a light color, which is spoken of as "pale."

For purposes of calculation, the viscosity of a liquid may be defined [1] as the force which will move a unit area of plane surface, with unit speed relative to another parallel plane surface, from which it is separated by a layer of the liquid of unit thickness. This absolute viscosity, expressed in cgs units, is called by the somewhat abbreviated name of dynes per square centimeter, although strictly speaking 1 dyne/cm^2 is the unit of pressure. Except where otherwise noted, viscosities in this paper are expressed in Poises in accordance with the suggestion of Deeley and Parr.[2] The reciprocal of the Poise is the unit of fluidity. Table 1 gives other units of viscosity, with references to articles in which they occur, and the required conversion factors.

TABLE 1

Units of Viscosity

1 dyne-second per square centimeter= 1 Poise [3]
1 pound-minute per square inch= 4.137×10^6 Poises [4]
1 kilogram-second per square meter= 98.07 Poises [5]
1 poundal-second per square inch= 2143 Poises [6]

Unfortunately none of the units of Table 1 have been generally employed by oil refiners and jobbers, although, according to Denton,[7] tests for viscosity have been in use for over 30 years. In the oil industry in the United States viscosity is usually measured by an instrument called a viscosimeter or viscometer, which permits a certain volume of oil to discharge through a short capillary tube, the "viscosity," so called, being reported as the time of discharge in seconds, a value which is of little significance unless the make of instrument is stated and is well known.

With the Engler viscosimeter, which is the instrument most commonly used in Germany, it is customary to express viscosi-

[1] Clerk Maxwell, Scientific Papers, 2, p. 7, 1890: O. Reynolds, Phil. Trans. R. S. 177, pt. 1, p. 164, 1886.
[2] R. M. Deeley, and P. H. Parr, Phil. Mag., 26, p. 87, 1913.
[3] A. C. McLaughlin, Jour. Am. Soc. Mechanical Engrs, July, 1914, p. 263.
[4] M. D. Hersey, Trans. A. S. M. E., 37, p. 167; 1915.
[5] R. Camerer, Zeit. für das gesamte Turbinenwesen, 1907, p. 461.
[6] W. K. Lewis, Jour. of Ind. and Eng. Chem., 8, p. 627; 1916.
[7] J. E. Denton, Trans. Am. Soc. Mechanical Engrs, 9, p. 369; 1887–88.

ties in "Engler degrees," which is a purely arbitrary scale found by dividing the time of discharge for the liquid being tested by the time for distilled water at a temperature of 20° C (68° F).

III. FORMULA FOR DETERMINING VISCOSITY BY CAPILLARY-TUBE EXPERIMENTS

Poiseuille[8] undertook about 1838 an experimental investigation of the flow in capillary tubes, and found that the discharge increased as the first power of the pressure and as the fourth power of the diameter of the tube and was inversely proportional to its length. Subsequent investigators have confirmed this result mathematically, and found that if μ represents the viscosity

$$\frac{Q}{t} = \frac{\pi P d^4}{128 \mu l} \tag{1}$$

where Q = discharge in time t.

P = difference in pressure between the two ends of the tube

l = length of tube, of diameter d

It had been noted by Poiseuille that although his results were usually concordant and, as is now known, agreed with equation (1) they did not agree with this law when the velocity was very high in his shortest tubes. Equation (1) would give fairly accurate results for a very long tube, but requires correction for use with most viscosimeters, since part of the pressure is in reality expended in setting the liquid in motion and not in overcoming viscous resistance. Hence a "kinetic-energy correction" must be deducted from P in order to get the effective pressure, and values of the viscosity, calculated from Poiseuille's formula, equation (1), will always be too high.

As pointed out by Couette,[9] the measured length of tube l requires correction to allow for "end effects" other than the loss of head causing acceleration. Making the necessary correction to P, as well as to l, equation (1) may be written [10]

$$\frac{\mu}{\gamma} = \frac{\pi g d^4 t}{128 Q (l+\lambda)} \left(h - \frac{m v^2}{g} \right) \tag{2}$$

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[8] J. L. M. Poiseuille, Mémoires de l'institut, savants étrangers, 9, pp. 433–544; 1846.

[9] M. Couette, Annales de chimie et de physique, 21, p.469; 1890.

[10] This is equivalent to equation (1a) of E. C. Bingham, Scientific Paper No. 278, Bureau of Standards, p. 319; 1916.

where μ = viscosity in Poises

d = diameter of tube, in centimeters

$(l+\lambda)$ = effective length of tube, in centimeters

Q = discharge (cubic centimeters) in time t, seconds

v = mean velocity in the tube, in centimeters per second

m = coefficient of the kinetic energy correction

g = acceleration of gravity = 981 cm/sec.^{2}

h = head in centimeters of liquid of density γ in grams per cubic centimeter

In deriving equation (2) from equation (1) the pressure, P, is expressed by $g\,\gamma\,h$ since in short-tube viscosimeters, such as are used for technical purposes, the pressure is usually produced by a head of the viscous liquid.   The expression $\frac{\mu}{\gamma}$ is known as the kinematic viscosity, and it is evident that in order to determine true viscosity with an efflux viscosimeter, densities must be determined by an auxiliary instrument.   In most viscosimeters the discharge takes place under a decreasing head, and the determination of the average head is not always a simple matter. Proper values for m and λ must also be determined, from theory or experiment, before equation (2) can be used.   It is usual to assume that λ is proportional to the radius of the tube, so that

$$2\,\lambda = nd \tag{3}$$

where n is a constant to be determined.

IV. REYNOLDS'S FORMULA FOR CRITICAL VELOCITY

It has been found that after a certain velocity of flow is exceeded and turbulence appears, equation (2) will no longer apply, and after an intermediate régime of rather unstable conditions, a further increase of velocity will produce a second stable régime of turbulent or hydraulic flow.   If the viscosity of water at 0° C (32° F) is taken as 0.017921, it may be calculated from data given by Reynolds [11] that when turbulence begins " Reynolds's criterion " will have the value

$$\frac{vd\gamma}{\mu} = 2010. \tag{4}$$

This value, which is usually taken as 2000, is only an approximation, and applies only to pipes of considerable length, as used by Reynolds.   This investigation has shown that it does not at all hold for short tubes.

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[11] O. Reynolds, Phil. Trans. R. S., **174**, pt. 3, p. 948; 1883.

According to Flowers [12] the principal types of technical viscosimeters were devised, shortly after the publication of Reynolds's work, so that with the standard, water, the velocity would fall just below the critical value, as determined by equation (4). He says, further, that Reynolds's results have been "taken erroneously" to mean that below this velocity no eddies would occur, even with short tubes. Hayes and Lewis [13] also find that the critical velocity is exceeded with most of the viscosimeters in common use.

V. METHODS OF FINDING THE RELATION BETWEEN KINEMATIC VISCOSITY AND TIME OF DISCHARGE

1. ARCHBUTT AND DEELEY'S EQUATION

For any given viscosimeter, when a constant volume, Q, is allowed to run out, equation (2) becomes

$$\frac{\mu}{\gamma} = A\ t - \frac{B}{t} \tag{5}$$

in which A and B will be constants if λ, m, and h are constant. Omitting the separate term for the kinetic-energy correction, Archbutt and Deeley [14] write

$$\mu = k\ \gamma t \tag{6}$$

where k, the ratio of kinematic viscosity to time of discharge, is a variable to be found by experiment. If the kinetic-energy correction is negligible, as with very viscous oils, k is a constant, and it follows from equation (6) that if two liquids with viscosities of μ_1 and μ_2 have times of discharge, t_1 and t_2, respectively, then equation (6) becomes simply the Ostwald formula

$$\mu_1 = \frac{\gamma_1\ t_1}{\gamma_2\ t_2}\ \mu_2 \tag{7}$$

It will be noted that equation (7) can not be derived from equation (5) except on the assumption that the kinetic-energy correction is negligible, and very serious errors are frequently caused by the use of equation (7) when this assumption is not warranted.

Archbutt and Deeley give tables of viscosities and densities of solutions of glycerol and water for use in finding the variable, k, for any given viscosimeter. Having obtained experimentally the times of discharge for several solutions, for the instrument in ques-

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[12] A. E. Flowers, Proc. A. S. T. M., 14, pt. 2, pp. 577, 582; 1914.
[13] H. C. Hayes and G. W. Lewis, Jour. A. S. M. E., 38, p. 629; 1916.
[14] L. Archbutt and R. M. Deeley, Lubrication and Lubricants, p. 180; 1912.

tion, values of k may be calculated from equation (6). Interpolated values of k may then be used to find the viscosities of other liquids.

2. HIGGINS'S GRAPHICAL METHOD

Higgins [15] gives a method for finding values for the constants in equation (5). By combining equations (5 and 6)

$$k = \frac{\mu}{\gamma t} = A - \frac{B}{t^2} \qquad (8)$$

Then if values of k are plotted as ordinates against values of $\frac{1}{t^2}$ as abscissas, the points lie on a straight line. The intercept of this line on the ordinate where $1/t^2 = 0$ gives the value of A, and the slope of the line gives the value of B. [16]

Speaking of the Redwood viscosimeter, Higgins says: "The value of A is, however, obtained more accurately from the slope of the curve connecting μ/γ and t, for values of t greater than 200. For these values B/t is negligibly small compared with $A\,t$." In this case the kinematic viscosity is proportional to the time, as in equation (7). It is evident that the value of 200 seconds applies only to the Redwood viscosimeter, and that for any other instrument a decision would have to be made as to the appropriate value.

3. UBBELOHDE'S EQUATION

Ubbelohde's tables [17] are computed from the formulas

$$Z = 4.072\,E - \frac{3.513}{E} = \frac{4.072\,t}{t_w} - \frac{3.513\,t_w}{t} \qquad (9)$$

$$\mu = 0.01797\,Z\gamma \qquad (10)$$

where E is a measure of viscosity known as "Engler degrees," which is obtained by dividing the time of discharge, t, for 200 cubic centimeters of the liquid to be tested, by the time, t_w, for distilled water at 20° C (68° F). t_w, expressed in seconds, is known as the water rate. The constant 0.01797 is taken as the viscosity of water at 0° C (32° F).

Concerning the "empirical" formula, equation (9), Ubbelohde says: "The formula was derived and confirmed by comparative

[15] W. F. Higgins, Collected Researches, The National Physical Laboratory, 11, p. 12; 1914.
[16] For methods of determining the equation of a curve which will fit a series of experimentally determined points, see C. T. Brady, jr., Engineering News, 63, p. 163, 1913; W. H. Herschel, Bull. Soc. for the Promotion of Engineering Education, 5, p. 33, 1915; J. B. Kommers, The Wisconsin Engineer, 20, p. 106; 1915.
[17] L. Ubbelohde, Tabellen zum Englerschen Viskosimeter, pp. 9, 27; 1907.

tests with a capillary apparatus which gave specific viscosity directly," specific viscosity being defined as $Z\gamma = z$. No detailed account of these tests appears to have been published.[18] Hayes [19] says that Ubbelohde's formula "is only claimed to be approximate," but it has been generally accepted as at least the best formula available for converting Engler time into Poises.[20] Ubbelohde [21] himself admits elsewhere that his equation does not apply to liquids of greater fluidity than water, while maintaining that it was "derived theoretically but has been experimentally confirmed." Speaking of liquids with a viscosity, expressed in Engler degrees, of 1 or less, he says:

For such thinly fluid substances almost identical Engler values are obtained for liquids which in reality have very different viscosities, * * * viscosities can not be deduced from the Engler values for such thin liquids. (See my article Die Zahigkeit des Leucht-petroleums und ein Apparat zu ihrer Bestimmung, Zeitschrift "Petroleum," 4, No. 15.)

It is therefore evident that Ubbelohde's formula does not apply when the viscosity is below a certain value which must be determined by further experiments. Assuming, however, that equations (9 and 10) apply to water at 20° C (68° F), the kinematic viscosity is given by the formula

$$\frac{\mu}{\gamma} = \left(\frac{4.072\,t}{t_{\mathrm{w}}} - \frac{3.513\,t_{\mathrm{w}}}{t}\right)0.01797 \qquad (11)$$

Since the water rate is a constant for any given instrument, equation (11) is seen to have the form of equations (2 and 5) and it is perhaps to this fact that Ubbelohde refers when he says that equation (9) has been derived "theoretically."

As shown in the first section of Appendix 1, equations (2 and 11) may be used to find what values of m and n would have to be assumed in equation (2) in order to agree with equation (9). The value for m, thus found, is 0.8862, which is a check on Meissner's value of 0.8989 for a similar but not identical constant, f.[22] The value of n is found to be 1.30, or somewhat less than the value of 1.64 proposed by Higgins. The most probable values of m and n for the Engler instrument will be discussed later, together with the cause of the inaccuracy of the Ubbelohde formula.

[18] L. Ubbelohde, Chem. Ztg., 31, p. 38; 1907.

[19] H. C. Hayes, in discussion of paper by Hayes and Lewis, loc. cit., p. 632.

[20] G. B. Upton and A. E. Flowers, Proc. A. S. T. M., 15, pt. 1, pp. 296, 317, 1915; P. C. McIlhiney, Jour. of Ind. and Eng. Chem., 8, p. 433, 1916; W. F. Higgins, loc. cit.; also The Petroleum World, June and July, 1913.

[21] L. Ubbelohde, Gen. Elec. Rev., 18, p. 968, (translated from the German journal "Petroleum"); 1915.

[22] W. Meissner, Chemische Revue über die Fett- und Harz- Industrie, 17, pp. 202–209; 1910. He uses equation (2) with λ=o, and the coefficient, m (which he calls f) is chosen "so that μ for water will have the correct value."

4. MEISSNER'S COMPARISON OF THE ENGLER AND THE SAYBOLT UNIVERSAL VISCOSIMETERS

If k_E and k_S are values of the variable k of equation (6) for the Engler and Saybolt [23] viscosimeters, respectively, then, when the kinetic-energy correction is negligible

$$\frac{\mu}{\gamma} = k_E \, t_E = k_S \, t_S \tag{12}$$

from which
$$\frac{t_E}{t_S} = \frac{k_S}{k_E} = \text{constant} = K \tag{13}$$

Meissner [24] gives the time of discharge for the Saybolt and Engler instruments, for liquids of five different viscosities, as in Table 2.

TABLE 2

Relative Times of Discharge for the Engler and Saybolt Viscosimeters, According to Meissner

Liquid	Reduced time, Engler, average of two instruments	Time, Saybolt	K
	Sec.	Sec.	
Water at 20° C (68° F)	50.94	28.55	1.784
Rape oil at 50° C (122° F)	233.6	169.1	1.381
Machine oil at 50° C (122° F)	357.0	258.6	1.381
Rape oil at 20° C (68° F)	730.8	515.6	1.417
Machine oil at 20° C (68° F)	2491.0	1759.0	1.416

It will be seen from the last column, which was not given by Meissner, that the minimum value of K does not occur for the maximum viscosity as it would according to Meissner's formulas for the two instruments. It is therefore necessary to attribute the discrepancy to experimental error, to surface tension or to some other cause not taken into account in deriving these equations. If it is assumed that the kinetic-energy correction is negligible in all Meissner's tests with oil, then, excluding the test with water, the average value of K is 1.399. As shown in the second section of Appendix 1, Meissner's method of averaging leads to a value of $K = 1.391$, and with the help of this value, and Meissner's

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[23] When the Saybolt viscosimeter is referred to in this paper, the Saybolt Universal is meant, and this should not be confused either with the earlier forms of Saybolt instrument (see W. M. Davis, Friction and Lubrication, p. 44, 1904), nor with the latest form with a cover (U. S. Patent No. 1132621, of Mar. 23, 1915). Much that is said concerning the Saybolt instruments which have been used in this investigation may not apply to the form with a cover.

[24] W. Meissner, loc. cit., 19, p. 30; 1912.

equation for the Engler instrument, it may be found that for the Saybolt instrument $n=1.21$ and $m=0.88$. These are not essentially different from the values previously found from Ubbelohde's equation for the Engler instrument. It will be noted that in spite of Meissner's disregard of n in his equation for the Engler instrument his own equations are equivalent to taking n into account for the Saybolt instrument.

VI. GRAPHICAL METHODS FOR STUDYING THE LAWS OF FLOW THROUGH CAPILLARY TUBES

1. GRÜNEISEN'S DIAGRAM

Grüneisen[25] has investigated the conditions under which Poiseuille's simple formula, equation (1), holds good with an error of not over a designated amount. He finds that for any series of tests with a given tube the velocity which will cause a given percentage error in the viscosity is

$$v = \frac{\mu}{d\,\gamma} f\left(\frac{l}{d}\right); \qquad (14)$$

or, in other words, the error depends merely upon two quantities, Reynolds's criterion, $\dfrac{vd\,\gamma}{\mu}$, and the ratio of length to diameter of tube. This latter ratio determines the error due to n, this constant depending upon the diameter and being independent of the length. It follows that if Reynolds's criterion is plotted against $\dfrac{l}{d}$ and the percentage departure from Poiseuille's law recorded for each point, lines of constant error may be drawn.

The viscosity in Poises, which must be known in order to calculate the percentage error, is ordinarily determined by a long-tube viscosimeter. When a series of tests has been made with such an instrument, with a slightly higher pressure for each successive run, it is usually found that the viscosity, calculated from equation (1) or equation (2), has a constant value for the lower pressures where the error due to velocity is negligible. This constant value is taken as the true viscosity. If it is found that there is a regular decrease in the apparent viscosity, as the pressure is decreased, the value approached, as the velocity approaches zero, must be considered as the true viscosity. If equation (2) is used, the fact that the apparent viscosity did not vary with the velocity

[25] E. Grüneisen, Wissenschaftliche Abhandlungen der Physikalisch Technischen Reichsanstalt, 4, pt. 2, p. 152; 1905.

would, if the kinetic-energy correction were appreciable, be an indication that the assumed value of m was sufficiently accurate. The assumption in regard to λ could not be checked without the use of a second tube.

The apparent viscosity, as calculated from equation (1), will be called μ^1 to distinguish it from the true viscosity μ. Then the percentage error in the viscosity as calculated by equation (1) will be

$$\left(\frac{\mu^1}{\mu} - 1\right)100 = \text{percentage error} \tag{15}$$

2. A NEW DIAGRAM [26]

In Fig. 1 Reynolds's criterion, $\dfrac{vd\,\gamma}{\mu}$, has been plotted against the percentage error in the Poiseuille formula, $\left(\dfrac{\mu^1}{\mu} - 1\right)100$. An advantage of this form of diagram is that, without the use of logarithms, straight lines are obtained, from which values of m and n may be derived. It has the advantage, as compared with Higgins's method, of retaining Reynolds's criterion as a warning when the velocity is too high for equation (2) to apply. If a line on Fig. 1 represents a series of tests on a single tube, with velocities all below the critical value so that equation (2) will apply, then, as shown in the third section of Appendix 1, the tangent of the angle between this line and the axis of abscissas will have the value

$$\tan \theta = \frac{32\,l}{100\,md} \tag{16}$$

and the value of the percentage error, at the point where the calibration curve, extended, intersects the axis of abscissas, is equal to

$$\left(\frac{l+\lambda}{l} - 1\right)100 = \frac{100\lambda}{l} = \frac{50\,nd}{l} \tag{17}$$

If, as maintained by Couette, λ always has a plus value, all calibration curves must intersect the axis of abscissas to the right of the origin. It follows from equation (16) that a straight line on Fig. 1 indicates that m does not vary with the velocity.

VII. APPLICATIONS OF THE NEW DIAGRAM

Fig. 1 shows points calculated from tests of Poiseuille and Holde with water of given temperatures.[26a] As shown in the fourth section

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[26] For the first published mention of this diagram, see Jour. of the Wash. Acad. Sciences, **6**, p. 155; 1916.
[26a] See Mitteilungen aus den Königlichen technischen Versuchsanstalten, Ergänzungsheft 1, p. 9; 1895.
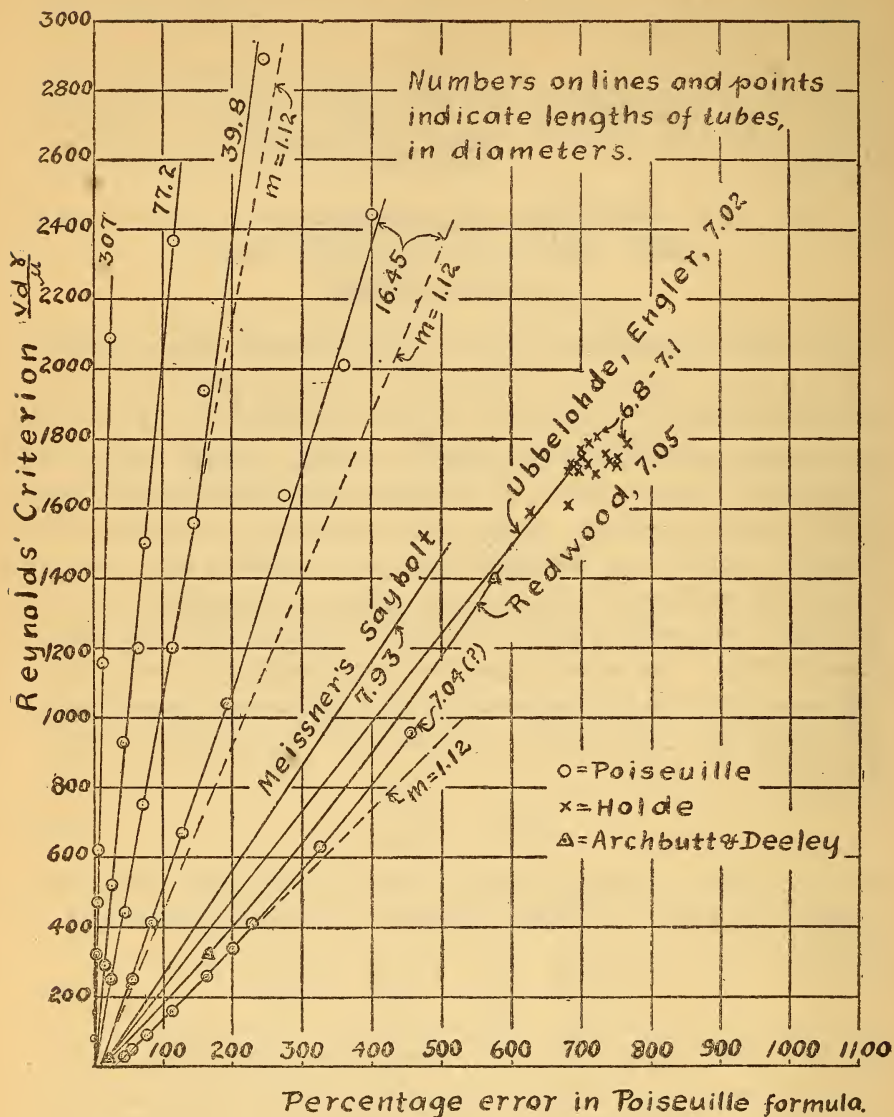
Fig. 1.—*A new diagram for studying the laws of flow through capillary tubes*

Most of Poiseuille's tests agree very closely with his formula. The excessive errors shown in Figs. 1 and 3 are due to the selection of tests of high velocity, to which, as recognized by Poiseuille, the formula does not apply.

of Appendix 1, the work of calculation may be simplified by using constants which depend on the dimensions of the tube, while true viscosities and densities may be taken from Appendix 2. Poiseuille made his experiments with glass tubes of various lengths, while Holde used Engler instruments with tubes of abnormal dimensions, so that the lengths varied from 6.8 to 7.1 diameters, the normal value being $\frac{2.00}{0.285} = 7.02$. Archbutt and Deeley's calibration of the Redwood viscosimeter was made with glycerol solutions, for which they give the viscosities and densities. The lines for the Engler and the Saybolt Universal viscosimeters are calculated from equation (9) and equation (36) of the second section of Appendix 1 by assuming values of the time of discharge. It will be noted that the upper end of the line for the Engler viscosimeter agrees fairly well with the average of the points calculated from Holde's tests.

Archbutt and Deeley's tests, when plotted on Fig. 1, show a curved line, but this curve is based on too few points to be conclusive. Poiseuille's shortest tube, A–7, had a ratio of length to diameter of $\frac{1 \ (?)}{0.1417} = 7.04$, and tests on this tube also give a curved line. Other tests of Poiseuille, with longer tubes, give lines which are straight at their lower ends. No definite conclusion can be drawn from Fig. 1 as to whether the curvature of the lines for Poiseuille's tube, A–7, and for the Redwood viscosimeter, is due to experimental error or is correct. While the question mark, which is Poiseuille's, shows that the length of the tube, A–7, is probably in error, only the position of the line on the diagram, but not its shape, would thereby be affected.

One object of this investigation was to determine whether tests on short tubes, such as are used in technical viscosimeters, could be correctly represented by straight lines on a diagram similar to Fig. 1, or, in other words, whether m and n of equation (2) are constants.

VIII. CONDITIONS AFFECTING THE ACCURACY OF MEASUREMENTS WITH TECHNICAL VISCOSIMETERS

In using the Engler and the Saybolt Universal viscosimeters, the following sources of error have been noted:

1. STARTING THE FLOW

In the Saybolt instrument the flow is started by removing a cork from the bottom of a tube which is concentric with and

extends below the lower end of the capillary tube. When the cork is inserted and the oil container is filled, the oil is prevented from flowing out by the compressed air inside the larger tube between the cork and the lower end of the capillary tube. If the cork leaks, as may easily happen, especially with very mobile liquids such as gasoline or hot water, this space will become filled and its contents will fall into the flask or upon the table as soon as the cork is removed. It should be noted that this liquid has not passed through the capillary tube during the measured time and that consequently it is a source of error if any of it falls into the flask. The flask usually supplied with the Saybolt instrument has a mouth $1\frac{1}{2}$ inches in diameter, and the use of a smaller diameter (for example, $\frac{7}{8}$ inch, as in the tests here described) would tend to reduce this error.

The cork should not be inserted while the liquid is flowing, as in this case also some liquid is trapped in the space above the cork. To wait for all the liquid to flow out, in case of a retest, causes considerable delay if the liquid is very viscous, but this difficulty may be avoided by using a glass rod to cover the upper end of the capillary tube; afterwards, however, inserting the cork, removing the rod, and using the cork to start the flow.

With some oils considerable trouble was experienced by the deflection of the jet from its vertical direction, so that it touched the outer tube, into which the cork is inserted, and left the instrument more nearly horizontally than vertically. Apparently this was due to a drop of a rather gummy oil forming a connection between the jet and the walls of the outer tube.

2. STOPPAGE OF THE OUTLET TUBE

Since the diameter of the capillary tube of the Saybolt instrument is less than that of the Engler it might be expected that more difficulty would be experienced with stoppage. If the time of discharge for the Saybolt instrument were too large, due to stoppage, and the Engler outlet tube were clear, the ratio of time, Engler, to time, Saybolt, would be too low, and there is reason to believe that this occurred not infrequently. While the plunger furnished with the Saybolt instrument is convenient for blowing out the capillary tube, it hardly compensates for the much greater inaccessibility of the tube as compared with that of the Engler instrument. It was always easy to see that the Engler tube was clear or to get at it to clean it. Advocates of the Saybolt instrument suggest the apprehension that splinters from the wooden

skewer used to stop the upper end of the Engler capillary tube might become lodged in the tube, but no difficulty was experienced on this account and I believe the fear to be groundless. It is possible that with years of use the tube might become enlarged near the upper end from wear, but this could be detected by the change in the water rate. On the other hand, the anxiety that the Saybolt outlet tube might be stopped was always present, although relieved by swabbing it out with a thread wound around a broom straw, and noting that the time of discharge was not changed. To avoid danger from lint, paper was always used to clean the oil container of the Engler instrument.

3. ERROR IN FILLING

With the Saybolt instrument the oil container is filled until it overflows into a gallery and then the gallery is emptied by using a pipette. With the Engler instrument the oil level is adjusted until three sharp points, equally spaced around a circle, just pierce the surface of the liquid. These gage points are similar to what is known to hydraulic engineers as a "hook gage" which is used to measure the height of water above a weir where extreme accuracy is desired. From experience with both methods, if care is taken, I believe more accurate filling of the oil container may be obtained with the Engler instrument, but with careless work the error would be greater than with the Saybolt instrument unless, indeed, the blunder was made of forgetting to empty the gallery or not removing enough of the liquid with the pipette. Very viscous oils cling to the sides of the gallery and may flow down and fill the gallery too full after it has been partly emptied. On the other hand, with the Engler instrument a very viscous oil will adhere to the skewer, thus increasing the amount by which the liquid surface is lowered when the skewer is withdrawn. An electric lamp connected to an extension cord was found convenient to get a better light on the gage points when filling the Engler instrument.

4. ERROR DUE TO BUBBLES IN THE MEASURING FLASK

The direction is usually given to let the jet fall into the flask without touching the neck, because otherwise there is an error due to the time required for the oil to flow from the point where it strikes the neck to the graduation. With many oils, however, this method of procedure gives rise to bubbles, which may cause a greater error, due to uncertainty as to the instant when the meniscus reaches the graduation. All things considered, I believe it is

1392°—17——3

preferable to avoid the bubbles by letting the jet strike the neck when necessary, which is fortunately not apt to be the case with a very viscous oil. Any error that may be caused by the flow of oil down the neck may be decreased by the selection of flasks which have the graduation near the top of the neck.

5. CONTROL OF TEMPERATURE

With the Engler and the older forms of Saybolt instruments the constant-temperature bath is heated only by a Bunsen burner. By fastening the Engler burner to a leg of the tripod with a piece of wire, so that it could be slipped up and down but could not swing and overheat the capillary tube or break the flask, the Engler burner was much more convenient to use than the Saybolt, as the temperature of the bath could be adjusted by the position of the burner as well as the size of the flame. On this account, and because the burner could not be considered an essential feature of a viscosimeter, the Saybolt burner was discarded entirely and the Saybolt bath was heated by placing it on the Engler tripod.

It is evident that if the bath was heated no higher than the temperature at which the oil was to be tested, the time required for the oil to heat up would be excessive, and that from practical considerations it is necessary to have the bath at a slightly higher temperature. The oil being tested will therefore increase in temperature during the run, and the best result will be obtained if the average temperature of the oil during the run is the temperature desired. With the Engler instrument the thermometer may be left in place throughout the run, and the temperature at the end is as easily noted as at the beginning, though perhaps not so accurately. But where the thermometer has to be withdrawn when the run is started, the temperature at the end of the run can only be estimated from the rate of rise of temperature before the thermometer was withdrawn. To get the average temperature during the run allowance must be made for the time lost in withdrawing the thermometer and in emptying the gallery before the run is begun.

It was found that there was a tendency for the temperature of the oil to increase when tests were made at room temperature, and this was doubtless due to the heat of the hand applied to the flask. It was therefore found necessary either to work at a temperature sufficiently above the room temperature so that more heat was lost by radiation than was gained in this manner or to

avoid surplus heat by keeping the temperature of the bath below that at which tests were made.

Since viscosity is not proportional to temperature, there is an inaccuracy in averaging temperatures. Two methods have been found convenient in keeping the temperature changes as small as possible. When the oil has reached the desired temperature the bath may be cooled to the temperature of the oil by siphoning off part of the water and refilling the bath with cold water. If other means of heating are available, it is more convenient to keep the bath as near as possible to the temperature desired and heat the oil outside the viscosimeter. It is necessary to allow for a slight fall in temperature when the oil is poured into the viscosimeter.

6. METHOD OF VISCOSIMETER COMPARISON WITHOUT TEMPERATURE CONTROL

With very mobile liquids the greatest error was in starting and stopping the watch, but it was assumed that this was practically eliminated by taking an average of 10 runs. This rule was adhered to for all tests except those with mixtures of kerosene and spindle oil used in obtaining Figs. 9 and 10. It was found that with the most careful work the difference between the maximum and minimum values of the time of discharge was rarely less than one second for each minute of the average time, which would bring extreme values slightly less than 1 per cent from the average.

With the more viscous oils the greatest error was due to difficulty of temperature control, a difficulty which became the more serious with the more viscous oils, which changed in viscosity the more rapidly with the temperature. Therefore I used a method which would give correct results, provided that the temperature was either constant or varied uniformly. Three viscosimeters, two Saybolt and one Engler, were placed side by side on the bench and the baths emptied so that the instruments were practically all in one air bath at room temperature. Then tests were made in the order Saybolt No. 580, Engler, Saybolt No. 727; Saybolt No. 727, Engler, Saybolt No. 580, which gave two tests for each instrument, and the average temperature for each instrument would be the same, provided that the length of time lost and spent in operations was always the same, and the rate of change of temperature of the room was uniform.

IX. THE RELATION BETWEEN KINEMATIC VISCOSITY AND TIME OF DISCHARGE FOR THE ENGLER AND THE SAY-BOLT UNIVERSAL VISCOSIMETERS

1. SUITABLE LIQUIDS

The number of easily obtainable liquids of which the viscosity is accurately known is very limited. Rayleigh[27] says: "The viscosity of water may be diminished (to about one-fourth) by heating it, or increased by admixture of alcohol." Bingham and Jackson[28] recommend water, and both alcohol and sucrose solutions. Redwood[29] proposed, and later adopted rapeseed oil as a standard, though, as pointed out by Meissner, there is considerable doubt as to whether even the best refined oil will not suffer considerable change in viscosity with time. Archbutt, in discussing Redwood's proposal, said that in a number of tests with a certain viscosimeter he found that "the viscosity varied from 100 to 109, whilst one or two samples went as high as 127 on the same scale. These samples were all genuine by chemical tests, and he supposed the oil which gave the high results to be summer rape, which is said to be more viscous than winter rape." Presumably any other vegetable oil would be of uncertain viscosity, both on account of variations in viscosity between different samples when fresh and on account of changes in viscosity of any one oil with time.

Long-tube viscosimeters are usually completely inclosed so that there can be no loss of the liquid being tested by evaporation. With instruments which discharge into the air evaporation is a serious source of error at high temperatures if solutions or mixtures are used which change in viscosity upon the loss of the more volatile constituent. Alcohol and sucrose solutions are both open to this objection, but it is to be noted that the viscosity of an alcohol solution of approximately 50 per cent alcohol does not change very rapidly with a change in concentration.

There is a great need of an easily obtainable liquid having a viscosity greater than 0.024 Poise, which is the value for a 50 per cent alcohol and water solution at a temperature of 25° C (77° F). Of the 72 chemical compounds investigated by Thorpe and Rodger[30] only 6 fatty alcohols have a viscosity greater than the 50 per cent solution of ethyl alcohol and water, the highest value

[27] Lord Rayleigh, Report British Advisory Committee for Aeronautics, **1**, p. 38; 1909–10.
[28] E. C. Bingham and R. F. Jackson, Scientific Paper, Bureau of Standards, No. 298; 1917.
[29] B. Redwood, Jour. Soc. for Chem. Industry, **5**, p. 127: 1886.
[30] T. E. Thorpe and J. W. Rodger, Phil. Trans. R. S., **185**, pt. 2, p. 572: 1894.

recorded being 0.14179 Poise for dimethyl ethyl carbinol at zero degrees C (32° F). The highest viscosity measured by Bingham and Jackson for a 60 per cent sugar solution at 11.61° C (52.9° F) was $\frac{1}{0.9075} = 1.102$ Poise. For a calibrating liquid of greater viscosity castor oil is sometimes used, for which Archbutt and Deeley [31] give a viscosity of 2.729 Poises at 100° F (37.8° C). The highest viscosity given by them is 11.156 Poises for "extra L. L." cylinder oil at the same temperature. MacMichael [32] suggests for a standard testing fluid "a high-grade neutral mineral oil of high viscosity, such as some of the medicinal oils now on the market," the viscosity of this oil to be determined by the long-capillary-tube method and the oil to be supplied by some "recognized authority."

2. DIMENSIONS OF INSTRUMENTS AND METHODS OF MAKING ADJUSTMENTS

The tests which are to be described were all made with three Saybolt Universal and one Engler viscosimeter. These instruments had been previously used by Waidner,[33] together with other instruments. The principal dimensions of the Saybolt Universal viscosimeter, according to Meissner and Gill, are given in Table 3, which also contains dimensions of the Engler viscosimeter used in this investigation, with the normal dimensions of the Engler instrument according to Ubbelohde for comparison.

TABLE 3

Dimensions of Saybolt Universal and Engler Viscosimeters

SAYBOLT UNIVERSAL

	Meissner, No. 752	Gill
Mean diameter of outlet tube...........................centimeters..	0.178	0.180
Length of outlet tube...do....	1.41	1.30
Length of outlet tube, in diameters.........................do....	7.93	7.22
Diameter of oil container.....................................do....	2.968	3.000
Head on lower end of tube, at start........................do....	12.688	12.60
Head on lower end of tube, at end..........................do....	4.016	....
Average head..do....	7.538	....
Capacity of container........................cubic centimeters..	....	70

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[31] Loc. cit., p. 190. See also Kahlbohm and Räber, Abh. d. Kaiser Leop. Carol deutsch akad. der Naturforschungen, 84, p. 203, 1905; Proc. A. S. T. 15, pt. 1, p. 302, 1915.

[32] R. F. MacMichael, Jour. A. S. M. E., 38, p. 1003; 1916.

[33] C. W. Waidner, Proc. A. S. T. M., 15, pt. 1, p. 284; 1915.

TABLE 3—Continued

Dimensions of Saybolt Universal and Engler Viscosimeters—Continued

ENGLER

	No. 2204 U	Ubbelohde	
		Normal	Tolerance
Mean diameter of outlet tube, top.........centimeters..	0. 2925	0. 290	±0. 002
Mean diameter of outlet tube, bottom.............do....	0. 2829	0. 280	±0. 002
Length of outlet tube.............................do....	2. 00 or 1. 97	2. 00	±0. 01
Length of outlet tube in diameters................do....	6. 94 or 6. 83	7. 02	....
Diameter of oil container.........................do....	10. 584	10. 60	±0. 1
Head on lower end of tube, at start...............do....	5. 184	5. 200	±0. 05
Head on lower end of tube, at end................do....	2. 911	a 2. 934	....
Average head....................................do....	3. 939	a 3. 959	....
Capacity of container...............cubic centimeters..	247	Not specified.	Not specified.

a According to Meissner.

It was stated that on account of unevenness of the top edge of the capillary tube in Saybolt instrument No. 765 the measurement of the mean length may be in error by 0.03 or 0.04 cm. It was found that the tubes of Saybolt instruments Nos. 580 and 727 have very nearly the same dimensions, but that instrument No. 765 has a considerably larger and longer tube. On this account it was considered that this instrument was probably not so close to the standard instrument, with which all others are compared by Mr. Saybolt, and hence less work was done with this instrument than with the other two. It will be noted that the length of the capillary tube, as given by Gill, does not agree very closely with the length as given by Meissner.

Table 4 shows the results obtained with liquids of known viscosity. Normal volumes were discharged, i. e., 200 cc with the Engler and 60 cc with the Saybolt instruments. The discordance of readings in the case of water with the Engler instrument is due to the tendency for the water to swirl, unless it is absolutely still when the skewer is withdrawn. Ubbelohde [34] especially warns the operator when finding the water rate that "before the beginning of the test the water should stand perfectly still and should also be disturbed as little as possible in with drawing the skewer."

This difficulty is not encountered with the Saybolt viscosimeter, presumably on account of the much smaller diameter of the container.

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[34] Tabellen, p. 24.

TABLE 4

Time of Discharge for Liquids of Known Viscosity for Four Viscosimeters, With Normal Volumes Put in and Discharged

DISTILLED WATER

Temperature, °C	Engler No. 2204 U Time, seconds	Saybolt Universal					
		No. 580		No. 727		No. 765	
		Time, seconds	Time, ratio	Time, seconds	Time, ratio	Time, seconds	Time, ratio
3.0		32.1		32.0			
4.0	56.3						
5.0		31.6					
5.5	55.5						
6.0		31.2					
7.0		30.7		30.8			
8.0	55.1						
9.0	54.7						
10.0	54.2	30.3	1.788				
11.0		30.2					
12.0				30.1			
13.0	53.6	30.0	1.782	30.0	1.790		
14.0	53.4			*a* 29.7	*a* 1.791		
15.0	53.3	29.4	1.818	*a* 29.4	*a* 1.811		
16.0	53.0	29.3	1.811	29.2	1.817	28.6	1.850
17.0	52.4 / 53.1	29.4	1.788 / 1.810	29.4	1.788 / 1.810		
18.0	52.1 / 52.9 / 52.6	28.8	1.805 / 1.835 / 1.822	28.9	1.801 / 1.830 / 1.818		
19.0	51.9 / 52.0						
20.0	51.2 / 51.3	28.5	1.801	28.3	1.813	28.0	1.832
20.5	51.3						
21.0	51.1						
23.0		28.6					
24.0						27.5	
24.5	50.3						
25.0				27.8			
27.0				27.5			
27.5		27.6					
30.0	49.6						
31.0				27.1			
40.0		26.7					
50.0	47.7	26.2 / 26.8	1.822 / 1.788	26.4 / 26.1	1.810 / 1.830	25.8	1.848
75.0	46.2	25.6 / 25.7	1.801 / 1.801	25.4 / 25.7	1.819 / 1.799	25.2	1.833

a At 15°.

TABLE 4—Continued

Time of Discharge for Liquids of Known Viscosity for Four Viscosimeters, With Normal Volumes Put in and Discharged—Continued

OLIVE OIL

Temperature, °C	Engler No. 2204 U	Saybolt Universal					
		No. 580		No. 727		No. 765	
	Time, seconds	Time, seconds	Time, ratio	Time, seconds	Time, ratio	Time, seconds	Time, ratio
16.0	806.3	553.4	1.458	534.6	1.510		
20.0	625.3	436.9	1.432	434.8	1.438		
21.0	602.5	414.5	1.451	416.0	1.449		
24.0	497.3	341.0	1.459	353.4	1.410		
30.0	387.3	270.6	1.431	277.6	1.397		
37.8	278.0	197.3	1.408	200.0	1.389		
	280.9	192.6	1.460	192.7	1.459	193.0	1.455
50.0	193.8	130.2	1.488	132.2	1.465		
65.6	133.4	85.5	1.560	85.5	1.560	85.7	1.557
	134.8		1.575		1.575		1.573
75.0	115.4	76.9	1.500	76.7	1.505	67.4	1.712
	118.3	75.4	1.569	72.7	1.628		1.753
100.0	85.7	52.0	1.651	51.5	1.664		

70 PER CENT ETHYL ALCOHOL

Temperature, °C	Engler No. 2204 U	No. 580		No. 727		No. 765	
15.0	63.0			35.3	1.788		
20.0	60.7	34.0	1.788	34.0	1.788		
25.0	58.0						
30.0	56.4			31.0	1.821		
35.0	55.2						
40.0	53.8			29.2	1.838		

10 PER CENT ETHYL ALCOHOL

Temperature, °C	Engler No. 2204 U	No. 580		No. 727		No. 765	
10.0	58.5	33.0	1.773	32.8	1.782		
15.0	56.5	31.6	1.792	31.6	1.790		
20.0	55.5			30.4	1.822		

50 PER CENT ETHYL ALCOHOL

Temperature, °C	Engler No. 2204 U	No. 580		No. 727		No. 765	
10.0	62.3						
20.0	69.3	40.2	1.725	40.2	1.725		

60 PER CENT SUGAR SOLUTION

Temperature, °C	Engler No. 2204 U	No. 580		No. 727		No. 765	
10.0	632.4			443.5	1.428		
25.0	266.8			164.4	1.622		
30.0	214.4			130.6	1.646		
35.0	174.6						
40.0	156.0						
45.0	144.1			75.2	1.918		
50.0	118.3						
54.4	93.6			56.2	1.665		

TABLE 4—Continued

Time of Discharge for Liquids of Known Viscosity for Four Viscosimeters, With Normal Volumes Put in and Discharged—Continued

ISOBUTYL ALCOHOL

Temperature, °C	Engler No. 2204 U	Saybolt Universal					
		No. 580		No. 727		No. 765	
	Time, seconds	Time, seconds	Time, ratio	Time, seconds	Time, ratio	Time, seconds	Time, ratio
20.0		41.2		41.2			
30.0				36.1			
40.0				33.5			
20.0	26.32	70 cc put in and 60 cc discharged					
	22.10	60 cc put in and 50 cc discharged					
	8.70	60 cc put in and 20 cc discharged					
	17.90	50 cc put in and 40 cc discharged					
	11.34	45 cc put in and 25 cc discharged					
	9.06	45 cc put in and 20 cc discharged					

It will be noted that the water rate for Saybolt No. 765 is a little lower than for the other two Saybolt instruments, but that there is substantial agreement in the time of discharge for olive oil. Meissner has discussed mathematically the different methods of making the final adjustments in viscosimeters, by changing the head, as is done with the Redwood instrument; by reaming the capillary tube, as is probably the method adopted with the Engler; and by shortening the tube, as appears to be the method employed with the Saybolt instrument. He concludes that an error in the head may be compensated by a change in diameter of the tube, and vice versa, but that the length of the tube should be accurately specified, with suitable tolerances. It follows from this that if instruments are adjusted by shortening the tube, they will agree at one viscosity but not at another, and this seems to be confirmed by the tests of Table 4.

As no theoretical advantage has been claimed for the tapered capillary tube of the Engler instrument, it seems probable that it was designed with regard to ease of manufacture. This thought was suggested by experience with steam-turbine nozzles, which were brought to the exact diameter required by means of a tapered reamer. This has the advantage over a cylindrical reamer that the reamers do not have to be kept so nearly up to standard size, since when one becomes smaller with wear it merely needs to be thrust in farther. In making a capillary tube for a viscosimeter, a tube of the right taper, but longer than necessary, might be

prepared, and then this could be cut off at the right points so as to give a tube of the required length and correct diameters at the two ends.

3. HIGGINS'S METHOD

The constants of equation (8) could be determined by finding the time, t, for liquids of only two different viscosities. It is necessary, however, to know what are the limits of the validity of equation (8) and what liquids are suitable for purposes of test. Over 500 tests, or 5000 runs, were made to answer these questions.

A graphical method, such as Higgins's, has the advantage over a purely numerical method, in that tests which are evidently discordant may be rejected. As will be shown later, it has been found that for the Saybolt and Engler instruments, all tests should be rejected for which Reynolds's criterion has a value in excess of 800. In calculating this value it would be sufficiently accurate to use the normal value of the tube diameter. For other types of instrument the value of the criterion might vary, depending upon the ratio of length to diameter of outlet tube, but additional experiments are needed before the value of the criterion for the critical velocity can be predicted from the dimensions of the tube.

Fig. 2, for the Engler instrument, has been obtained from data of Table 4. If the usual practice is followed by using only tests with water at 20° C (68° F) and with one or more liquids of fairly high viscosity, the dotted line would be the result. It is evident, however, that this line is incorrect because it does not agree with tests with water at other temperatures or with alcohol and water solutions.

4. TENTATIVE EQUATIONS FOR THE ENGLER AND THE SAYBOLT UNIVERSAL VISCOSIMETERS

Figs. 3 and 4 have been calculated from data of Tables 3, 4, and 10. Two series of tests by Couette are also shown on Fig. 3. In both figures Reynolds's criterion is plotted against the percentage error in Poiseuille formula due to end effects, the necessary calculations being made as for Fig. 1.

Fig. 3 shows a break in the curves for both viscosimeters at points where the ordinate has a value of about 1000. Since the value of Reynolds's criterion, $\dfrac{vd\gamma}{\mu}$, calculated from tests with water at 20° C (68° F), is found to be about 1500 for the Saybolt and 1750 for the Engler instrument, as shown on Fig. 1, it follows that water at that temperature is not a suitable liquid for testing purposes, as also appeared from Fig. 2.

The dotted line in Fig. 3 for Ubbelohde's formula for the Engler instrument, is taken from Fig. 1. If equation (9) were correct throughout the entire range, the dotted line would coincide with
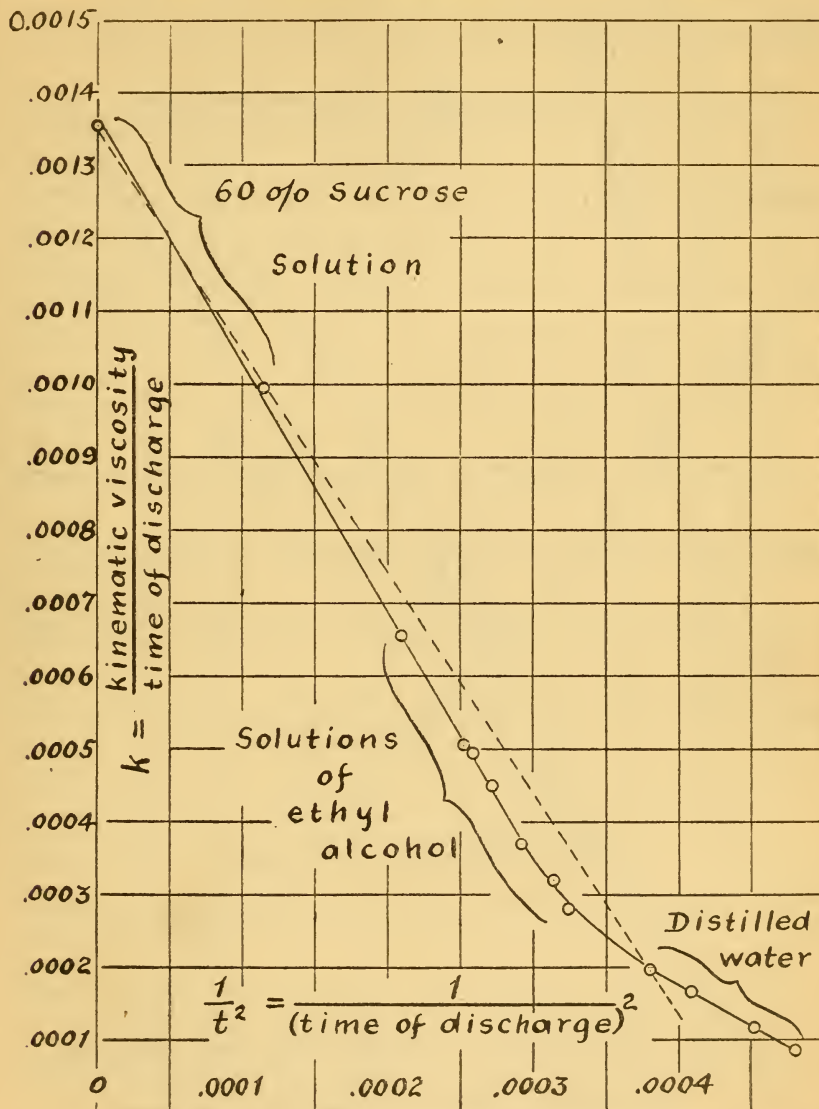
FIG. 2.—*Higgins's method for obtaining the ratio between kinematic viscosity and time of discharge for the Engler viscosimeter*

the curve for the Engler readings. The discrepancy between the two gives the magnitude of the error in using Ubbelohde's formula,

Fig. 3 shows that the greatest error in this formula will be for a liquid having a viscosity of about 0.015 (as for 10 per cent alcohol

solution at 20° C (68° F)) or for a value of Reynolds's criterion of about 1000, and that this error will amount to about 20 per cent. Ubbelohde's curve is fairly close to the true curve for values of
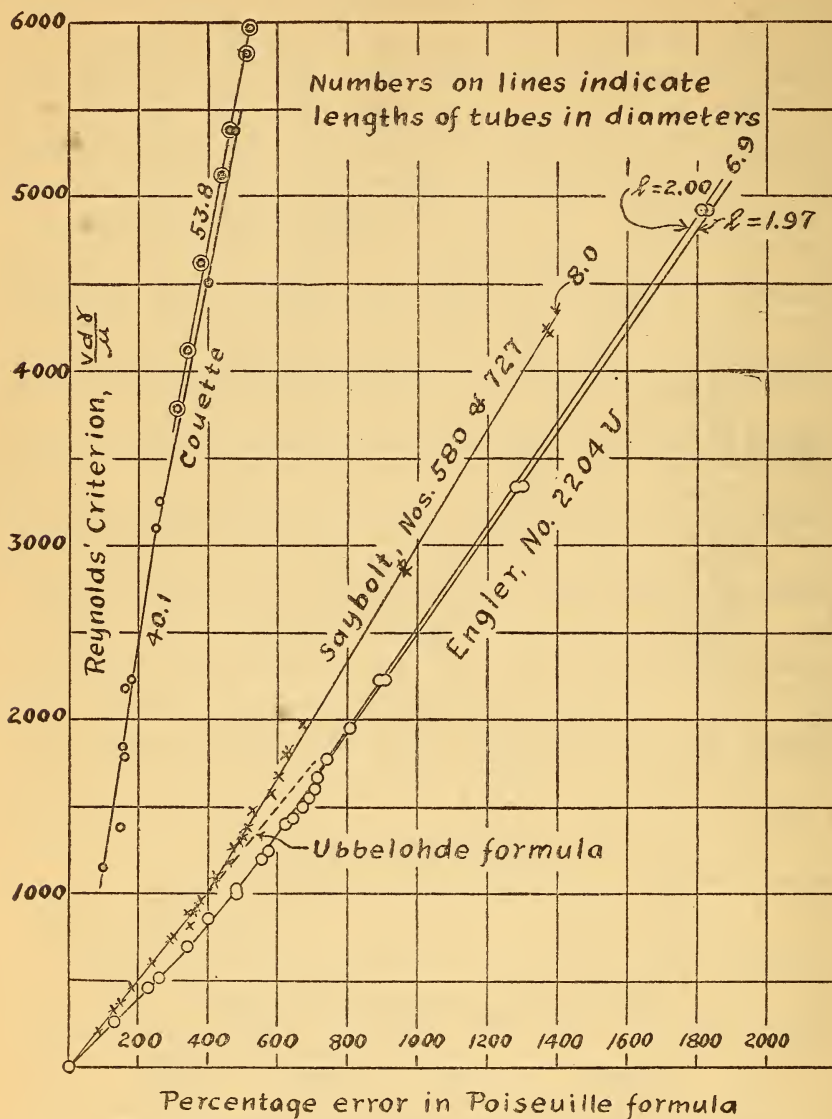
FIG. 3.—*Determination of critical velocity by the new diagram*

Reynolds's criterion of 1750 and zero, so that the error in his equation is mainly in the kinetic-energy correction. It is evident from equation (16) that the true value of m must be greater than 0.89, the value previously found from Ubbelohde's equation, pro-

vided equation (23) is used for calculating the average head, as was done in plotting these curves.

Tests with abnormal filling have been omitted from Figs. 3 and 4 so as not to obscure the diagrams, but by plotting to a larger
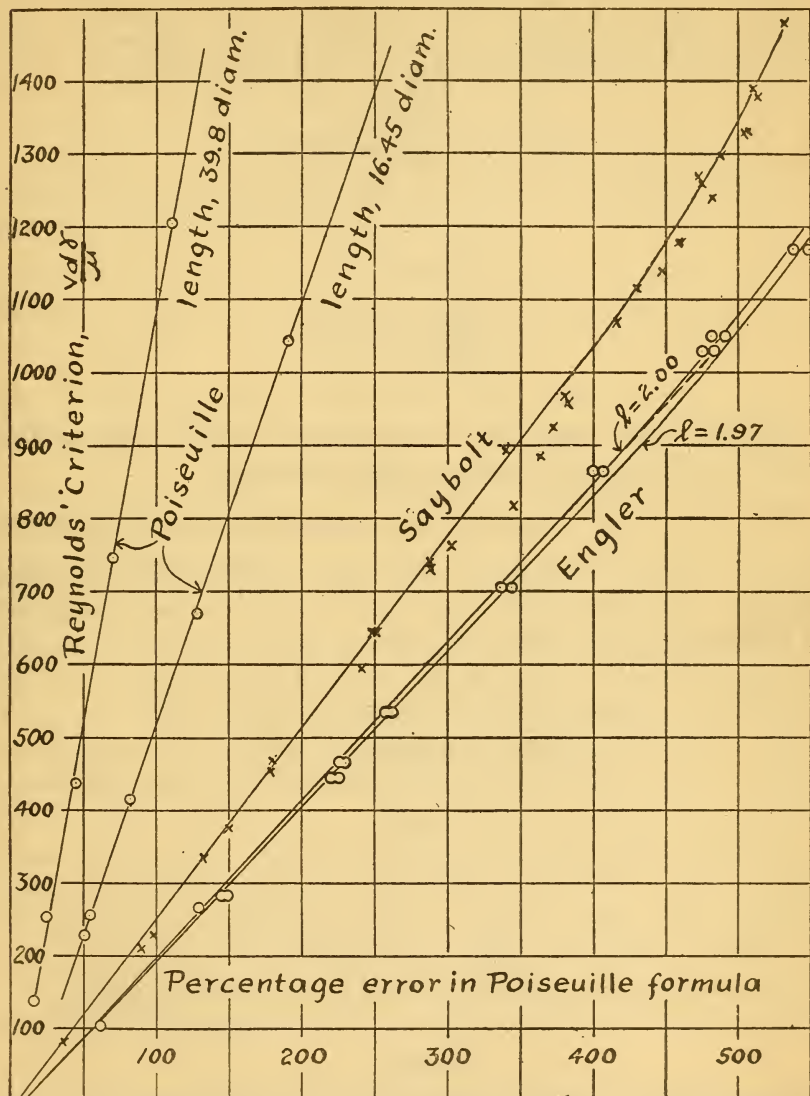
FIG. 4.—*Determination of equations for the Engler and the Saybolt Universal viscosimeters*

scale it was found that the more nearly constant the head, the nearer the points lie to the calculated curves for $m = 1.12$. That Boussinesq's value of 1.12 is correct, within the limits of experimental error, if the average head is correctly determined, is also

shown by Poiseuille's tests in Fig. 1, where the lines were drawn as calculated for $m = 1.12$, and are seen to agree with the experimentally determined points, except for the shorter tubes and for the higher velocities.

Values of n have been found graphically from Poiseuille's data, with the help of equation (17), and are given in Table 5. They show that n is probably variable and is always positive.

TABLE 5

Values of n from Poiseuille's Data

l/d	n
111.0	6.5
67.4	5.4
47.8	4.7
16.45	3.3
39.8	0.0
77.2	7.8

The last three values are probably less accurate because Poiseuille's readings were taken at higher velocities.

The value of $\dfrac{l+\lambda}{l}$, which has been selected for the Saybolt instrument as most in accordance with these considerations and with Fig. 4, is 1.05, from which values of 1.10 and 1.11 were computed for the Engler instrument with tube lengths assumed as 2.00 and 1.97, respectively, taking as the ratio of time, Engler, to time, Saybolt, the experimentally determined value of 1.45.

The above assumptions in regard to l and λ are equivalent to taking a value of n of 0.8 for the Saybolt instrument and of 1.3 and 1.5, respectively, for the Engler.

In order to find values of m from equation (16), readings were taken on Fig. 4 of the percentage error at points where Reynolds's criterion had a value of 1000, as given in Table 6. The value for the Engler instrument, with an assumed tube length of 1.97, is calculated from the percentage error for an assumed length of 2, as read from the dotted extension of the experimental curve.

TABLE 6

Percentage Error in the Poiseuille Formula When Reynolds's Criterion Has a Value of 1000

Instrument	Percentage error
Saybolt	385
Engler, tube length assumed, 2.00 cm	470
Engler, tube length assumed, 1.97 cm	478.7

It was found that m had a value of 1.03 for the Engler instrument and 0.97 for the Saybolt, and that the equations for obtaining kinematic viscosity would be

$$\frac{\mu}{\gamma} = 0.00213 \, t - \frac{1.535}{t} \text{ (Saybolt)} \tag{18}$$

$$\frac{\mu}{\gamma} = 0.00147 \, t - \frac{3.74}{t} \text{ (Engler)} \tag{19}$$

For the sake of comparison other equations which have been obtained for the same purpose have been collected in Table 7, dropping figures which are not significant.

<div align="center">

TABLE 7

Equations for Determining Absolute Viscosity from Times of Discharge of Short-Tube Viscosimeters

ENGLER VISCOSIMETER

</div>

Authority	Equation for kinematic viscosity
Calculated from Ubbelohde's equations	$0.00144 \, t - \frac{3.23}{t}$
Calculated from Meissner's equations	$.00157 \, t - \frac{3.58}{t}$
Flowers	$.00157 \, t - \frac{3.58}{t}$
Higgins	$.00144 \, t - \frac{3.22}{t}$
Upton	$.00143 \, t - \frac{3.22}{t}$

<div align="center">

SAYBOLT UNIVERSAL VISCOSIMETER

</div>

Calculated from Meissner's equations	$0.00219 \, t - \frac{1.497}{t}$
Flowers	$.00219 \, t - \frac{1.497}{t}$
Upton	$.00193 \, t - \frac{1.242}{t}$

<div align="center">

REDWOOD VISCOSIMETER

</div>

Calculated from Meissner's equations	$0.00260 \, t - \frac{1.561}{t}$
Flowers	$.00263 \, t - \frac{1.577}{t}$
Higgins	$.00260 \, t - \frac{1.715}{t}$
Upton	$.00232 \, t - \frac{1.387}{t}$

Examination of Table 7 shows that there are practically only two values given for the constants of equation (5) for the Engler instrument, Meissner and Flowers differing from the others because

they have used only one corrective factor, f, instead of the two factors n and m. Upton's equations for the Saybolt and Redwood instruments are different from those given by other authorities because based on different experimental data. The constants which I have calculated from Meissner's equations are the same as those calculated by Flowers, except for the Redwood viscosimeter. In regard to this instrument Flowers says that Meissner's constants do not check very well with test results and that he has therefore recalculated the constants.

5. EQUATIONS FOR INSTRUMENTS OF STANDARD DIMENSIONS

It must be remembered that equations (18) and (19) were derived from tests on certain instruments which happened to be available for experimental purposes, the dimensions for the Engler instrument being given in Table 3.

Ubbelohde,[35] in speaking of the report of the committee of the American Society for Testing Materials in 1914 on comparative data on the time of discharge of the Saybolt, Saybolt Universal, Redwood, and Engler viscosimeters, says:

> The investigation can not, however, be considered as finished, and will not be described in more detail here, as within a short time the standardization will be given out by the International Petroleum Commission in Karlsruhe (Drugs, Oils, and Paints, 26, Nos. 6 and 7, 1910: Petroleum, Berlin, 6, 2029, 1911.).

Waidner states that the Bureau of Standards has entered into negotiations with Mr. Saybolt with a view to defining the dimensions of the Saybolt Universal viscosimeter and to adopting suitable tolerances. Until this has been done equation (18) can not be corrected to apply to a viscosimeter of standard dimensions.

It will be seen from Table 3 that the diameter of the capillary tube of Engler viscosimeter No. 2204 U is a little too large, the head is a little too small, and the length of tube may be a little shorter than the mean prescribed length. The net effect of these deviations from the average standard dimensions has been calculated by Meissner's method, and it was found that the water rate should be 50.3 seconds. This value does not agree with the data of Table 4 for the Engler with distilled water, but it might be assumed that the higher water rate actually found was due to some roughness of the inlet, which would have an effect when the flow was in the hydraulic régime but would not appreciably increase the time of discharge in the régime of viscous flow. If, then, equation (19) is taken as applying to an instrument having

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[35] Gen. Electric Rev., loc. cit., p. 969.

a water rate of 50.3 seconds and is corrected in accordance with equation (9) so that it would apply to a normal instrument having a water rate of 51 seconds, it would become

$$\frac{\mu}{\gamma} = 0.00145 \, t - \frac{3.79}{t} \qquad (20)$$

while if it is assumed that the experimentally determined water rate of 51.3 seconds is due to causes which influence viscous as well as hydraulic flow the equation would be

$$\frac{\mu}{\gamma} = 0.00148 \, t - \frac{3.72}{t} \qquad (21)$$

In view of the many uncertainties involved, neither equation (20) or (21) can be regarded as much to be preferred to equation (19), and they are of interest mainly as showing to what extent a change of water rate will change the constants of equation (5).

X. APPLICATIONS OF THE NEW EQUATIONS FOR THE ENGLER AND THE SAYBOLT UNIVERSAL VISCOSIMETERS

1. RATIO OF KINEMATIC VISCOSITY TO TIME OF DISCHARGE

If A and B are taken from any equation of the form of equation (5), the kinematic viscosity may be calculated for any assumed values of t. The change of régime must not, however, be overlooked, and if values of the kinematic viscosity are desired for the hydraulic régime, they can not be calculated from equations (18) and (19). However, the value of the factor k of equation (8) changes so rapidly with a change of t for these low viscosities that accurate determinations can not be made with the Saybolt or Engler viscosimeters, no matter what equations are used. The curves of Fig. 5 have been calculated from equations (18) and (19) for values of the Engler time above 56 seconds and for Saybolt time above 31 seconds.

2. COMPARISON OF VISCOSIMETER READINGS AT STANDARD TEMPERATURES

Fig. 6 may be used for conversions between the different methods of expressing so-called viscosity, commonly in use in the United States, [36] for approximate estimates of the readings at commonly used temperatures, 100°, 130°, and 210° F having been recommended as standard temperatures by a committee of the American Society for Testing Materials, and 50° and 100° C

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[36] For other conversion diagrams, see Gill, loc. cit., p. 164–168; J. R. Battle, Lubricating Engineer's Handbook, p. 61, 1916; Society of Automobile Engineers Data Sheets, 2, p. 18, 1916.

being commonly employed with the Engler viscosimeter.[37]   Lower temperatures are sometimes used, but with oils with a paraffin base such determinations are of little significance on account of the influence of the varying condition of the paraffin.   Fig. 6 shows the rate of change of viscosity with change of temperature which may be expected, on the average, with oils having a paraffin base.   Oils with an asphalt base may show a higher rate, and compounded oils a lower rate of change.
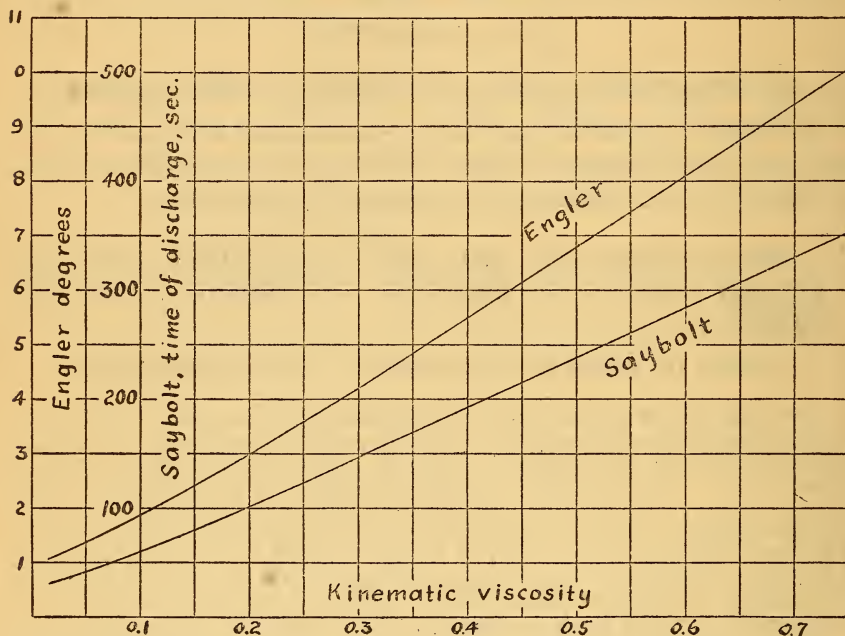
FIG. 5.—*Ratio of kinematic viscosity to time of discharge, as calculated from the new equations*

As an example of the use of Fig. 6, if an oil gives a value of 168 seconds, Saybolt time, at 130° F (54.4° C), by following the ordinate of this value it may be seen that this corresponds to a value of 4.8 Engler degrees at the same temperature.   Also if the oil is cooled to 50° C (122° F) the reading obtained with the Engler instrument would be 5.7° and the Saybolt reading would be 198 seconds.   At 100° F (37.8° C) the corresponding values would be 10.65 Engler degrees and 375 seconds, Saybolt.   On the other hand, if the temperature was raised to 210° F, at which, for the purposes of this approximate estimate, the viscosity is practically the same as at 100° C, the readings would be 1.6 Engler degrees and 50 seconds Saybolt.

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[37] D. Holde, Zeit. des Vereines deutscher Ingenieure, 56, pt. 2, p. 1461, 1912.
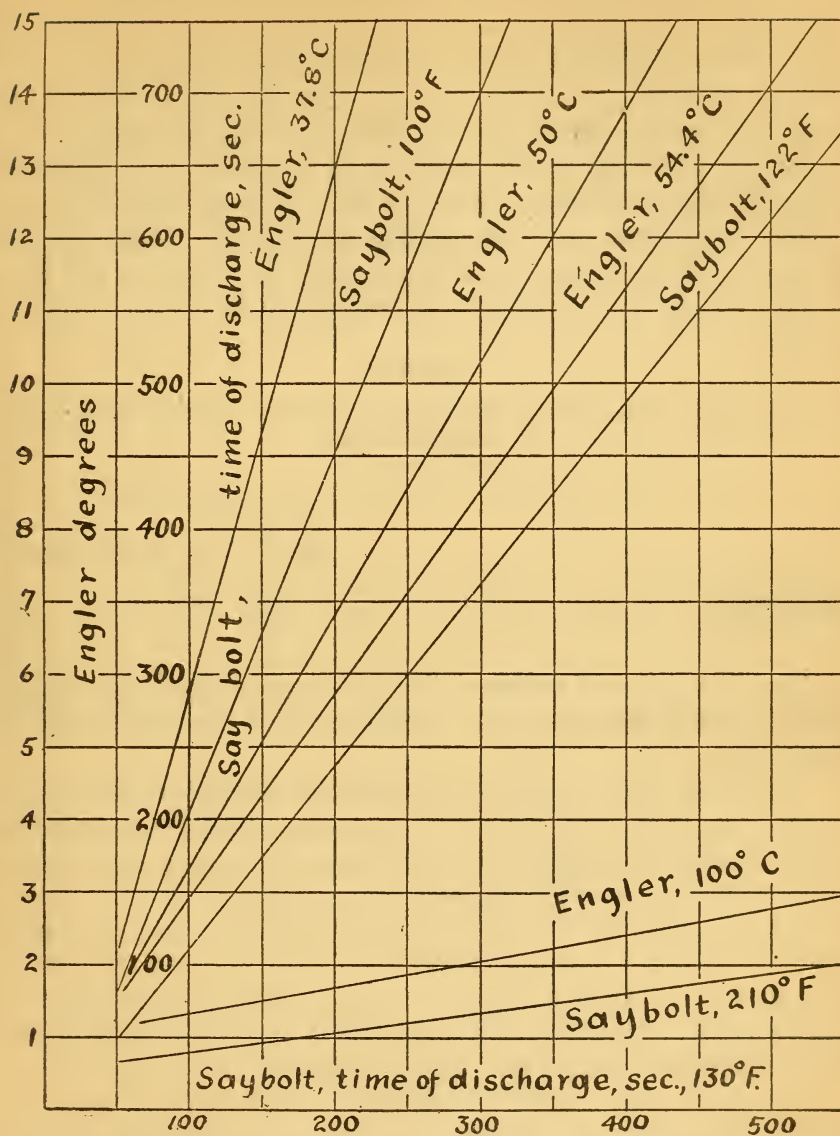
FIG. 6.—*Comparison of readings of the Engler and the Saybolt Universal viscosimeters at standard temperatures*

This diagram applies only to paraffin base oils, and should be used only for rough estimates.

XI. RATIO OF TIMES OF DISCHARGE OF THE ENGLER AND THE SAYBOLT UNIVERSAL VISCOSIMETERS

A question of great practical importance is the ratio of the times of flow of the Engler and Saybolt viscosimeters when the normal volumes, 200 and 60 cc, respectively, are discharged. For very viscous liquids, when the kinetic-energy correction is negligible, this ratio is a constant as by equation (13), assuming that the effect of surface tension or any other influence not considered in equation (2) is negligible or constant. Table 8 gives the time ratios for the Engler and Saybolt instruments, and also for the Engler and Redwood as determined by several investigators.

TABLE 8

Ratios of Times of Discharge for the Engler, Saybolt, and Redwood Viscosimeters for High Viscosities

Instruments	Ratio according to—			
	Meissner	Upton	Flowers	Higgins
Engler-Saybolt..	1.39	1.35	1.39	....
Engler-Redwood...	1.65	1.62	1.67	1.81

These values were obtained from the constants in their equations, omitting the last terms, which represent the kinetic-energy correction.

On account of the large experimental error which could not be avoided, even after the method of operation without temperature control had been adopted, it was necessary to make a great many tests, so that, even if the plotted points appeared like the milky way, the center line of the band would be close to the true value and the danger of showing nonexistent humps, as with the dotted line on Fig. 7, would be avoided. The results of these tests are shown in Fig. 7, together with those of Meissner and of committees of the American Society for Testing Materials.[38]

Taking the values of the time ratio for tests, where the time, Engler, has a greater value than 300 seconds, and where the ratio may be assumed nearly constant, 13 tests on Saybolt instrument No. 727 gave an average of 1.457, and 12 tests on Saybolt No. 580 gave an average of 1.444. The points at which the instruments begin to drip are indicated on Fig. 7, but they do not appear to

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[38] Proc., **10**, p. 142, 1910; **14**, pt. 1, p. 356, 1914.

correspond with any marked changes in the time ratio. If dripping begins at much lower viscosities than indicated, it would serve as a warning that the liquid was not running freely through the capillary tube.

Morgan [39] and Ronceray [40] have shown that the size and condition of the end of the tube have a marked effect on the formation of drops. This is the reason for prescribing the outside diameter of the capillary tube in the Engler instrument.
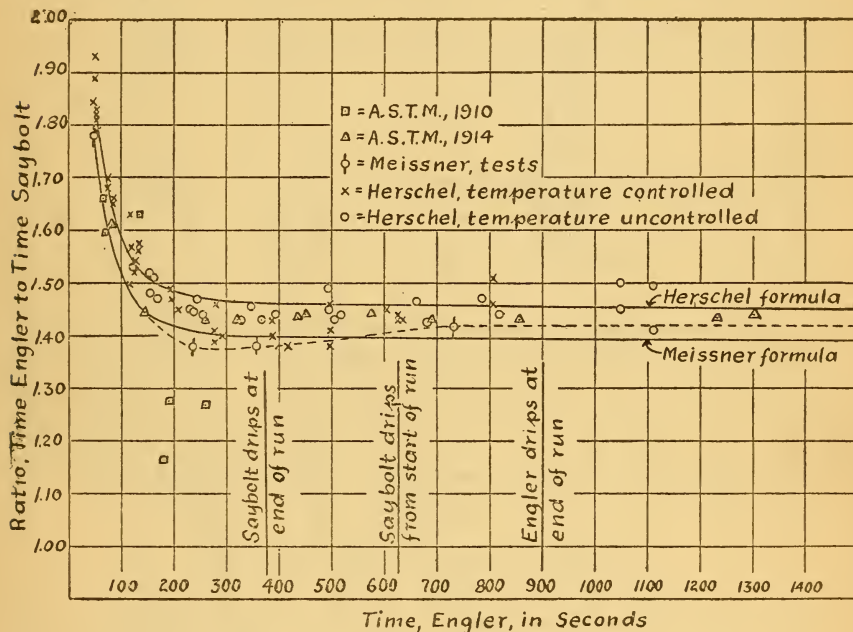
FIG. 7.—*Ratio of kinematic viscosity to time of discharge, as calculated from the new equations*

XII. METHODS OF SAVING TIME IN VISCOSIMETRY

1. ESTIMATION OF THE TIME OF DISCHARGE FROM THE NUMBER OF DROPS PER MINUTE

Advantage was taken of the longest runs to determine the relation between number of drops in the first and last minute and the time of discharge. The results are shown in Fig. 8, which should be used only for a very rough determination of viscosity on account of the variation in the size of a drop with the surface tension. It might serve, however, to inform the operator of a

[39] J. L. R. Morgan, Jour. Am. Chem. Society, 37, pt. 1, p. 1461, 1915.
[40] P. Ronceray, Annales de chimie et de physique, 22, p. 107, 1911.
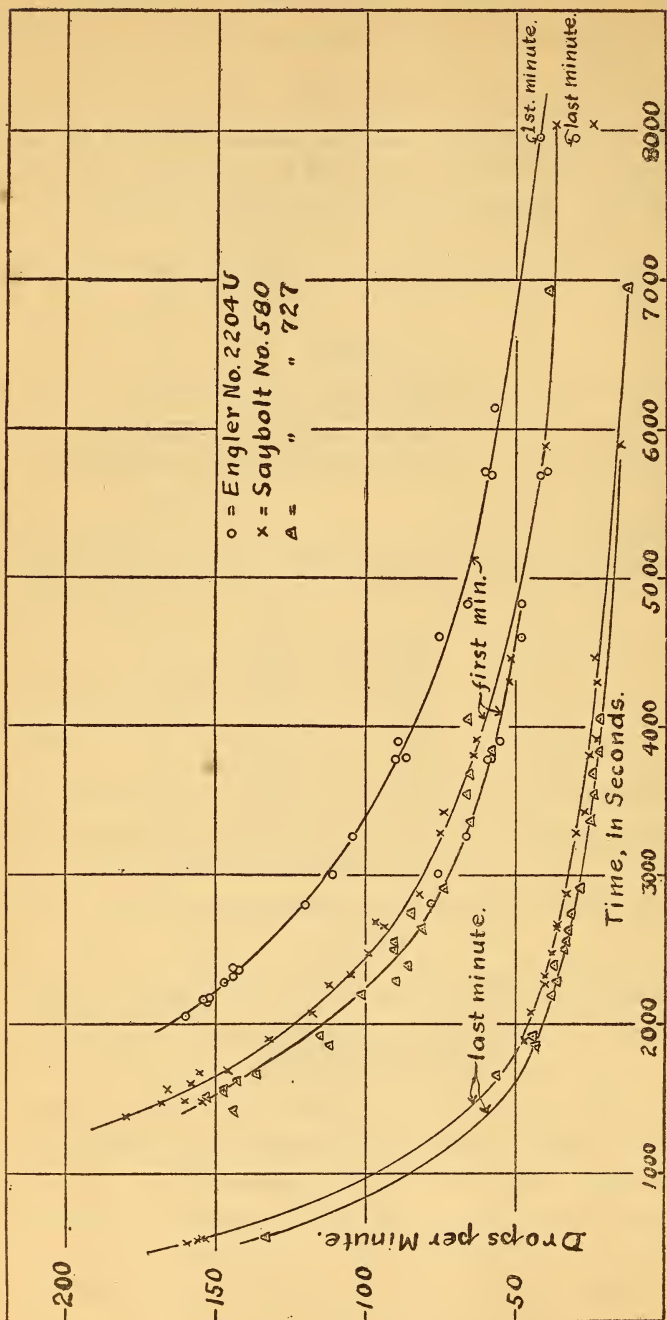
FIG. 8.—*Estimation of times of discharge of the Engler and the Saybolt Universal viscosimeters with normal filling, from number of drops per minute*

viscosimeter, after he had counted the drops in the first minute, whether he had time to attend to other work before again taking up his stop watch.

2. ESTIMATION OF THE NORMAL TIME OF DISCHARGE FROM THE TIME FOUND WITH ABNORMAL VOLUMES PUT IN AND DISCHARGED

Another method of saving time with the Engler instrument is to put more than the normal quantity of about 240 cc into the oil container so as to increase the head, or more commonly to decrease the time of discharge by using less than the normal volume. The latter method also makes it possible to use the Engler instrument when only a small amount of the liquid is available for test.

Holde [41] put in 45 cc of oil and ran out 20 cc, but gives no reason why he chose these amounts. He finds that the conversion factor necessary to get the normal time for 240 cc put in and 200 cc run out is 7.24, and considers that his results, which depart by at most 6 per cent from this average value, show a satisfactory agreement.

Ganz [42] undertook to check Holde, and also obtained conversion factors for other abnormal volumes put in and discharged, as shown in Table 9. Since the variations in Ganz's values are independent of the viscosity of the oil used in the tests, they may be considered due to experimental error.

TABLE 9

Conversion Factors Obtained by Ganz

Volume of oil put in (cubic centimeters)..............	45	45	50	60	120
Volume of oil discharged (cubic centimeters).........	20	25	40	50	100
Factor to change time of discharge to that for 240 cc put in and 200 cc discharged:					
Lowest...	6.91	5.36	3.46	2.64	1.60
Highest..	7.63	5.87	3.79	3.05	1.76
Average..	7.43	5.55	3.62	2.79	1.65

It is evident that conversion factors for any abnormal filling or volume discharged may be calculated if the average head can be determined. The average head, which must be used in equation (2), is not half the sum of the initial and final heads, $\frac{h_1 + h_2}{2}$ but can only be obtained from a formula based on an assumed relation

[41] D. Holde, Mittheilungen aus den Königlichen technischen Versuchsanstalten, 17, p. 62; 1899.

[42] L. Ganz, Chemische Revue über die Fett-und Harz-Industrie, 6, p. 218; 1899.

between head and time of discharge. If, as in hydraulics, it is assumed that $v = c\sqrt{2gh}$, and hence the time varies inversely as $\sqrt{h}$, the average head will be [43]

$$h = \left(\frac{h_1 - h_2}{2(\sqrt{h_1} - \sqrt{h_2})}\right)^2 = \left(\frac{\sqrt{h_1} + \sqrt{h_2}}{2}\right)^2 \tag{22}$$

Meissner bases his formula for average head on equation (1) and finds

$$h = \frac{h_1 - h_2}{\log_e h_1 - \log_e h_2} \tag{23}$$

This assumes time of discharge proportional to $1/h$. Since equation (23) was obtained on the assumption that the kinetic-energy correction could be disregarded, it evidently would not strictly apply to very fluid oils nor to water. Holde [44] investigated the effect of an excess or deficiency in the volume put into the oil container, but his variations in volume are not of sufficient magnitude to check the accuracy of equations (22) and (23). These equations have been used to calculate conversion factors for the heads used by Ganz and for other abnormal heads, and the results are given in Table 10. Instead of calling the normal filling 240 cc, as given by Ganz, the value of 247 cc has been used, because this was found to be the volume required to fill Engler viscosimeter No. 2204 U up to the gage points. The calculations were made for a cylinder with a flat bottom; that is, the dishing of the bottom of the oil container was ignored. The dishing was found to hold 31 cc, so that the calculations are in error in all cases in which the volume left in the container at the end of the run was less than that amount. It will be seen from the table that the conversion factors calculated from the various equations do not differ greatly.

Tests were made with a variety of oils using the abnormal fillings indicated in Figs. 9 and 10 in order to find to what extent Meissner's formula held good, and within what limits of viscosity. It was to be expected that it would cease to hold good when the kinetic-energy correction became appreciable, but that above a certain time of discharge the factor needed to find the time for the normal filling of 247 cc put in and 200 cc discharged would be

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[43] See J. T. Fanning, A Practical Treatise on Hydraulics and Water Supply Engineering, p. 221, 1893; M. Merriman, Treatise on Hydraulics, p. 70, 1916.

[44] D. Holde, Mittheilungen aus den Königlichen technischen Versuchsanstalten, Ergänzungsheft, **1**, p. 5; 1895.
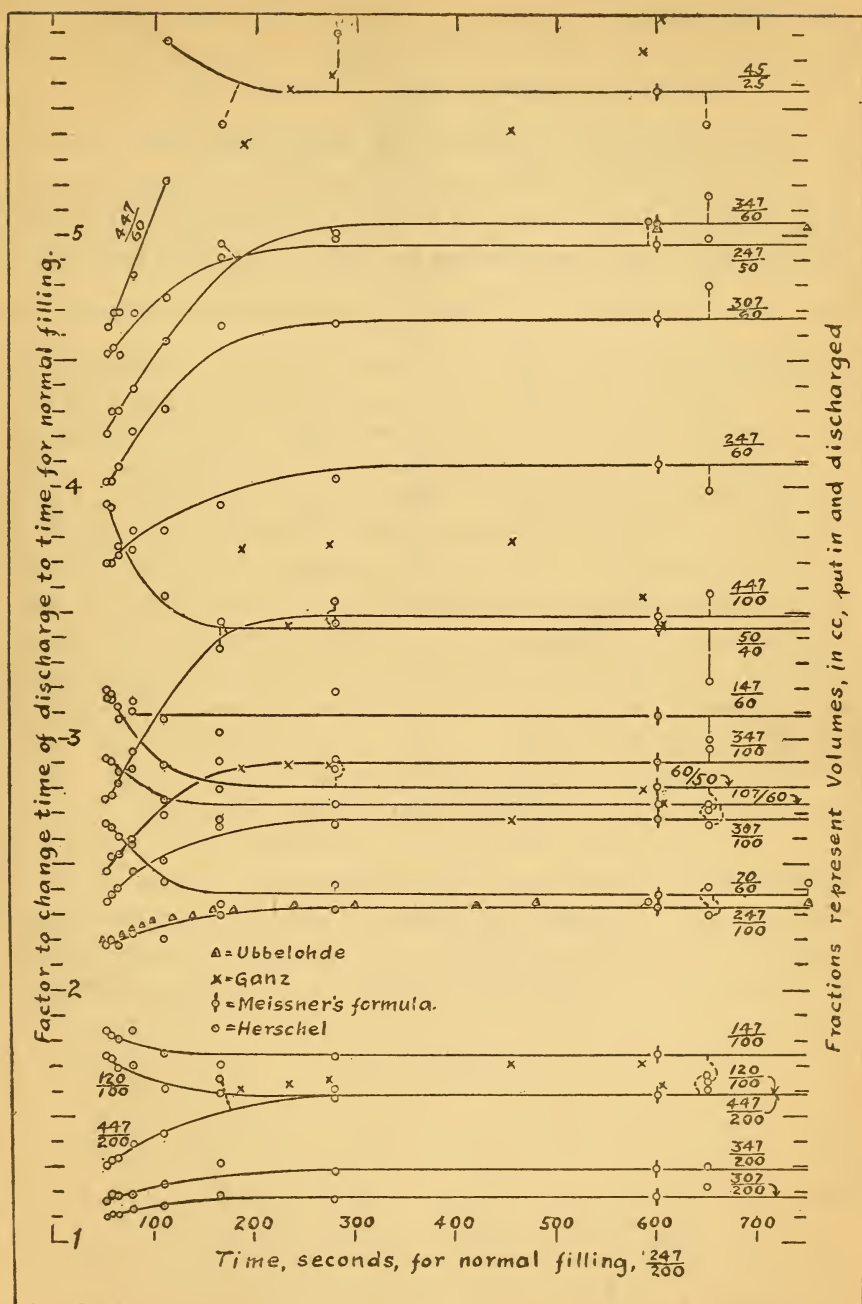
FIG. 9.—*Validity of Meissner's formula for average head, for the Engler viscosimeter with abnormal volumes put in and discharged*

a constant.   That this is the case is shown by Fig. 9, which indi-
cates that the factor may be considered constant for Engler times
of over 300 seconds with the normal filling.   In drawing the curves
of Fig. 9 the tests for liquids having an Engler time of less than

55 seconds and data for fillings of $\frac{60}{20}$ and $\frac{45}{20}$ have been omitted.
It will be noticed that the curves turn down at the left end when
more than 200 cc have been put into the oil container at the start,
and that they turn upward when less than 200 cc have been used.

The curves for fillings of $\frac{120}{100}$ and $\frac{447}{200}$ are interesting because the
calculated factor is practically the same for both.   In drawing all
curves it has been assumed that, starting from the left end, the
curves would rise or fall, as the case might be, until tangent to
the straight line indicating the calculated value of the factor.   This
procedure might be questioned in the case of those fillings in which
less than 31 cc are left in the container at the end of the run, but
has been adhered to even then on account of the lack of con-
cordance in experiments made with the more viscous oils, especially
with the smaller fillings.   Ganz evidently experienced the same
difficulty.   On the whole, it may be concluded that his conversion
factors should be used with considerable caution and that the
calculated values of Table 10 are in all probability much more
accurate, in cases where the Engler time is over 300 seconds.   For
less viscous oils, the conversion factor is best taken from Fig. 9 or
Fig. 10.   Ubbelohde [45] takes as the conversion factor when the
normal amount is put in and 100 cc discharged, a value of 2.353,
which he says is constant for Engler times of from 390 seconds
up and is less for more fluid oils.   When the normal amount is
put in and 50 cc discharged, he gives 5.030 as the conversion
factor.

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[45] Tabellen, p. 25.

TABLE 10

Calculated Conversion Factors for Engler Viscosimeter to Change Time of Discharge to Time for Normal Filling

Volume put in	Volume run out	Factor if average head equals half the sum of initial and final heads	Factor from Fanning's formula, equation (18)	Factor from Meissner's formula, equation (19)
cc	cc			
247	200	1.000	1.000	1.000
447	200	1.562	1.580	1.587
447	100	3.404	3.467	3.488
447	60	5.860	5.973	6.017
347	200	1.281	1.291	1.295
347	100	2.843	2.891	2.910
347	60	4.925	5.021	5.055
307	200	1.169	1.175	1.177
307	100	2.618	2.662	2.678
307	60	4.550	4.634	4.669
247	100	2.281	2.318	2.331
247	60	3.989	4.067	4.091
247	50	4.843	4.941	4.973
147	100	1.719	1.742	1.750
147	60	3.052	3.108	3.089
120	100	1.567	1.586	1.592
107	60	2.678	2.727	2.742
70	60	2.332	2.371	2.384
60	50	2.742	2.791	2.809
60	20	7.277	7.453	7.484
50	40	3.357	3.420	3.439
45	25	5.428	5.536	5.575
45	20	6.855	7.022	7.049

In the course of experiments here described use was made of isobutyl alcohol, of which a sufficiently large quantity (247 cc) could not be obtained to fill the Engler instrument to the gage points. Attempt was therefore made to get a probable value for the normal time by using abnormal fillings and making use of conversion factors. If Fig. 9 were employed to obtain these conversion factors, the calculations would have to be made by successive approximations, since the required factors depend upon the Engler time for normal filling, which is unknown. The factors were therefore plotted in Fig. 10 against the time of discharge for abnormal filling. Table 11 shows the method of estimating the normal time of discharge for 247 cc put in and 200 cc run out for isobutyl alcohol at 20° C (68° F). The second
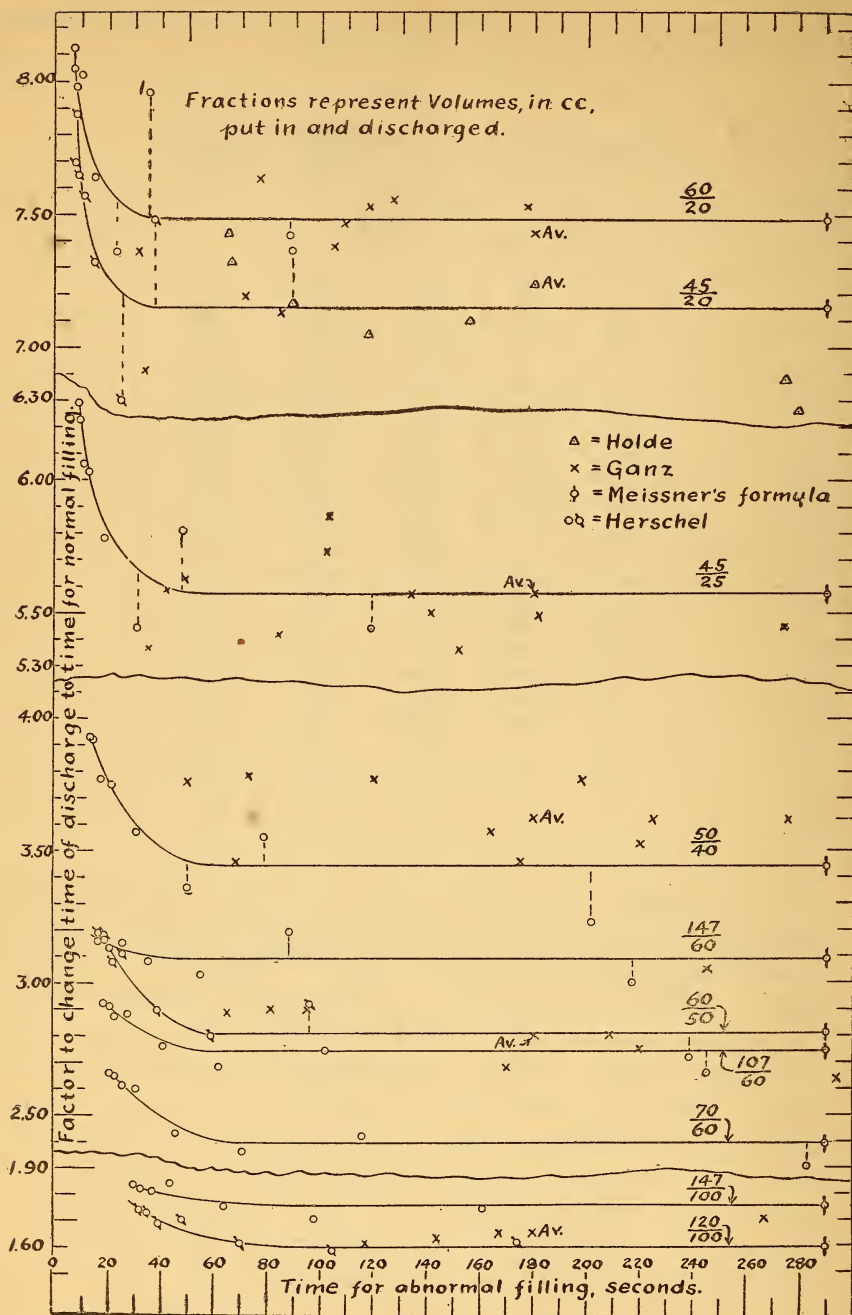
FIG. 10.—*Conversion factors for the Engler viscosimeter with abnormal filling*

point from the bottom, on the Engler curves of Figs. 3 and 4, is calculated from this estimated time.[46]

TABLE 11

Normal Time of Discharge of Isobutyl Alcohol, at 20° C, with Engler Viscosimeter

Volume put in	Volume discharged	Time	Factor from Fig. 10	Normal time of discharge
cc	cc	secs		secs
70..	60	26.32	2.61	68.6
60..	50	22.10	3.12	68.9
50..	40	17.90	3.78	67.7
45..	25	11.34	6.15	69.7
45..	20	9.06	7.70	69.7
60..	20	8.70	7.90	68.8
Average estimated time...........................				68.9

On account of the small capacity of the oil container of the Saybolt instrument only a few tests were made with the container only partly full. These tests are given in Table 12, the volume discharged being 30 cc in every test. Diagrams similar to Figs. 9 and 10 would be of little value for the Saybolt instrument, but these tests were made to show the effect of a more constant head.

TABLE 12

Times of Discharge for Saybolt Universal Viscosimeters with Abnormal Volumes Put in and Discharged

Liquid	Temperature	Time
	°C	secs
Viscosimeter No. 580, 44 cc put in, 30 cc discharged:		
Water..	3	18.36
Do...	3	18.46
10 per cent ethyl alcohol..........................	15	18.06
Isobutyl alcohol.................................	15	25.96
Viscosimeter No. 727, 40 cc put in, 30 cc discharged:		
Water..	3	18.56
Do...	3	18.64
10 per cent ethyl alcohol..........................	15	17.98
Do...	20	17.80
60 per cent sucrose solution.......................	25	107.86
Do...	54.4	32.58

[46] For the viscosity and density of isobutyl alcohol, see T. E. Thorpe and J. W. Rodger, Phil. Trans R. S., 185, pt. 2, p. 538, 1894; I. Pierre and E. Puchot, Ann. de Chim. et de Phys., 22, p. 306, 1871; E. C. Bingham and Miss J. P. Harrison, Zeit. für physikalische Chemie, 66, p. 1, 1909.

XIII. CONCLUSION

The conclusions from this investigation for short tubes, such as are used in the Engler and the Saybolt Universal viscosimeters, are as follows:

1. The value of the product of the velocity of flow and diameter of tube, divided by the kinematic viscosity, must not be greater than 800 if the flow is to be viscous rather than turbulent.

2. Consequently water is not a suitable liquid for use in finding the relation between viscosity and time of discharge, and Ubbelohde's equation and all others based upon it are seriously in error.

3. A small but positive amount must be added to the measured length of tube to get the effective length.

4. Boussinesq's value of 1.12 for the coefficient of the kinetic-energy correction would be approximately applicable if the average head could be correctly determined.

5. If the average head is determined by Meissner's formula, which is the best available, although unsatisfactory and far from accurate, a lower value must be used for the coefficient.

It is desirable that each operator of a viscosimeter should be able to obtain the relation between viscosity and time of discharge for his own instrument without use of the tube dimensions, which are difficult to determine with sufficient accuracy. Tables of data in regard to suitable liquids for this purpose are given in Appendix 2. If Higgins's method is used, it is not necessary to determine the tube dimensions.

WASHINGTON, April 14, 1917. .

APPENDIXES

Appendix 1.—UBBELOHDE'S AND MEISSNER'S EQUATIONS AND METHOD OF CALCULATING REYNOLDS'S CRITERION

1. VALUES OF m AND n FROM UBBELOHDE'S EQUATION

For water at 20° C (68° F), $t = t_w = 51$ seconds [47] and the kinematic viscosity, from equation (9), would be 0.010045 instead of the best available value of 0.010068. To correct for this discrepancy, the constants of equation (9) were multiplied by the factor 1.00501 and the viscosity of water at 0° C corrected, giving

$$\frac{\mu}{\gamma} = (4.0924 - 3.5306) \, 0.017921 = 0.010068 \tag{24}$$

or in general

$$\frac{\mu}{\gamma} = 0.073340 \, t/t_w - 0.063272 \, t_w/t \tag{25}$$

which, with $t_w = 51$, reduces to

$$\frac{\mu}{\gamma} = 0.0014380 \, t - 3.2269/t \tag{26}$$

Now equation (2) may be written in the form

$$(l + \lambda) \frac{\mu}{\gamma} = \frac{\pi \, d^4}{128 \, Q} \left(ght - \frac{16 \, Q^2 \, m}{\pi^2 d^4 t} \right) \tag{27}$$

in which the mean head, h, calculated from equation (19) was found to be 3.9583 cm (checking Meissner's value of 3.959 cm), $Q = 200$ cc, $g = 980.665$ cm per second,[2] $l = 2.0$ cm, and λ with the help of equation (3) may be written 0.1425 n, since $d = 0.285$ cm. Then equation (27) reduces to

$$(2 + 0.1425 \, n) \frac{\mu}{\gamma} = 0.0031428 \, t - \frac{7.9578 \, m}{t} \tag{28}$$

When the kinetic-energy correction is negligible, combining equations (26) and (28) gives

$$\frac{\mu}{\gamma} = 0.0014380 \, t = \frac{0.0031428 \, t}{2 + 0.1425 \, n} \tag{29}$$

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[47] Meissner gives the water rate as 50.94, but such exactness seems unwarranted, and Ubbelohde merely indicates by his tables that the time should lie between 50 and 52 seconds.

from which $n = 1.3021$. Substituting in equation (28) this value of n, with $t = 51$ when the kinematic viscosity is 0.010068, the value of m is found to be 0.88621.

Similarly, for Engler instrument No. 2204 U, for which the diameter of the oil container is 10.584 cm and the initial head on the lower end of the capillary tube is 5.184 cm, the avergae head was found to be 3.9392 cm. As regards the length of tube it was stated "the extreme length of the capillary tube was found to be 20.0 mm + 0.1 mm, while the length of the capillary measured inside between the points at the ends of the tube where the rounding begins was found to be 1.970 mm + 0.05 mm."

With a mean diameter of 0.287 cm, equation (28) becomes

$$(l + 0.1435\ n)\ \frac{\mu}{\gamma} = 0.0032158\ t - 7.9578\ m/t \tag{30}$$

It will be noted that the last term remains unchanged, being in fact simply $\dfrac{200\ m}{\pi\ 8t}$. Using the experimentally determined water rate of 51.3 seconds, equation (30) becomes

$$(l + 0.1435\ n)\ \frac{\mu}{\gamma} = 0.16497 - 0.15512\ m \tag{31}$$

while from equation (25)

$$\frac{\mu}{\gamma} = 0.0014296\ t - 3.2459/t \tag{32}$$

By combining equations (30) and (32) it is found that if

$$l = 1.97,\ n = 1.9474 \tag{33}$$

$$l = 2.00,\ n = 1.7382 \tag{34}$$

and then, with the help of equation (31), $m = 0.91748$ in either case, whether the length is 1.97 or 2 cm.

2. VALUES OF m AND n FROM MEISSNER'S EQUATION FOR THE SAYBOLT VISCOSIMETER

From the dimensions of Saybolt viscosimeter No. 752, given by Meissner, equation (35), which is of the same form as equation (30) of section 1, may be obtained.

$$(1.411 + 0.089n)\frac{\mu}{\gamma} = 0.0030368t - 2.3873m/t \tag{35}$$

Meissner gives for the Saybolt instrument the equation

$$t_S = 228.7 \frac{\mu}{\gamma}\left(1 + \sqrt{1 + \left[\frac{0.01309}{\left(\frac{\mu}{\gamma}\right)^2}\right]}\right) \tag{36}$$

where

$$\frac{\mu}{\gamma} = 0.08019\ E - 0.07013/E$$

$$= 0.0015723\ t_E - \frac{3.5767}{t_E}$$

E being the viscosity in Engler degrees. For a very viscous oil, when the kinetic-energy correction is negligible, equation (36) simplifies to

$$K = \frac{t_E}{t_S} = \frac{1}{228.7 \times 2 \times 0.0015723} = 1.3905 \tag{37}$$

and from a combination of equations (35) and (37), with equation (28) of section 1,

$$K = \frac{0.0030368(2 + 0.1425 n_E)}{0.0031428(1.411 + 0.089 n_S)} = 1.3905 \tag{38}$$

Since the value of n_E (i. e., n for the Engler instrument) is known, n_S is the only unknown quantity in equation (38), and it is found to be 1.2109. This leaves m the only unknown quantity in equation (35), if Meissner's value of $t = 28.55$ is taken for the water rate and the kinematic viscosity is equal to 0.010068 as in section 1. Substituting these values m is found to be 0.87791.

3. DETERMINATION OF m AND n FROM FIG. 3

From equations (1) and (2) and the definition μ'

$$\frac{\mu'}{\mu} = \frac{(l + \lambda) gh}{l(gh - mv^2)} \tag{39}$$

If, in Reynolds's criterion, the viscosity is expressed as in equation (8),

$$\text{Reynolds's criterion} = \frac{32v^2(l + \lambda)}{d(gh - mv^2)} \tag{40}$$

But from equations (1) and (2), when Reynolds's criterion is zero and the kinetic-energy correction is zero,

$$\frac{\mu'}{\mu} = \frac{l + \lambda}{l} \tag{41}$$

so that from equation (15) the percentage error will be $100\lambda/l$

under these circumstances. If the angle between the calibration curve of a capillary tube and the axis of abscissas is called θ then,

$$\tan \theta = \frac{\text{Reynolds's criterion}}{100\left(\dfrac{\mu'}{\mu} - \dfrac{l+\lambda}{l}\right)} \tag{42}$$

from which, with the help of equations (15), (39), and (40),

$$\tan \theta = \frac{32l}{100md} \text{ or } m = \frac{32\sqrt/d}{100 \tan \theta} \tag{43}$$

4. METHOD OF CALCULATING REYNOLDS'S CRITERION AND $\frac{\mu'}{\mu}$ FROM THE QUANTITIES CONTAINED IN EQUATION (6)

$$\frac{vd\gamma}{\mu} = \frac{4Qd\gamma}{\pi d^2 t\mu} = \frac{K_1\gamma\varphi}{t} \tag{44}$$

where φ is the fluidity, the reciprocal of the viscosity.

$$\frac{\mu'}{\mu} = \frac{\pi d^4 gh\gamma t}{128Ql\mu} = K'\gamma\varphi t = \frac{K'}{k} \tag{45}$$

K_1 and K^1 are constants for a given viscosimeter for a given volume put in and discharged, and unlike Archbutt and Deeley's factor, k, they do not vary with the velocity. With the help of viscosities given in Archbutt and Deeley's calibration table, page 180, values of γ may be interpolated from table on page 179, in order to get values of Reynolds's criterion from equation (44). In calculating $\frac{\mu'}{\mu}$ these values are not needed, as it is simpler to use values of $k = \frac{\mu}{t\gamma}$. The mean head was calculated from Meissner's formula, which, in order to save finding two natural logarithms, is preferably written.

$$h = \frac{h_1 - h_2}{\log_e\left(\dfrac{h_1}{h_2}\right)} \tag{46}$$

Appendix 2.—TABLES OF KINEMATIC VISCOSITIES AND OTHER PHYSICAL CONSTANTS OF LIQUIDS USED IN VISCOSIMETRY

Bingham and Jackson [48] have given tables of fluidities, viscosities, and densities of standard liquids for calibrating viscometers or viscosimeters. When using short-tube instruments, however, the kinematic viscosity, or viscosity divided by the density, is more often the physical constant required. Since it appeared that Bingham and Jackson calculated viscosities from

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[48] E. C. Bingham and R. F. Jackson, Scientific Paper No. 298, Bureau of Standards; 1917.

fluidities, the fluidities given in their tables have been used, in preference to the viscosities, in calculating the kinematic viscosities of Tables 13 and 14. Viscosities are given in centipoises (cp). Since the centipoise is one-hundredth of the cgs unit of viscosity and the fluidity is the reciprocal of the viscosity expressed in cgs units,

$$\frac{\text{viscosity, in cp}}{\text{density}} = \frac{100}{\text{fluidity} \times \text{density}} = 100 \times \text{kinematic viscosity}$$

In Tables 13 and 14, the densities of water for temperatures up to 41° C (105.8° F), were taken from Chappuis [49] and those for higher temperatures were taken from Thiesen. [49]

·Densities for the alcohol and water solutions of Table 13 were taken from the work of Osborne, [50] extrapolation being necessary for temperatures below 10° C (50° F) and above 40° C (104° F). The constants given by Osborne were used in making these extrapolations. The densities at 0° C (32° F) obtained in this manner are subject to error, due to the maximum in the density of water at 4° C (39.2° F). The magnitude of this error may be judged by comparing, in Table 13, the densities calculated from Osborne's data with those calculated from the tables of Landolt-Börnstein. [51] In view of the difference between the two authorities in the density of pure alcohol, no attempt was made to correct the extrapolated densities of the solutions to allow for the influence of the maximum in the density of water.

The extrapolated values for densities of solutions at high temperatures may be compared with the values ot Schwers, [52] while densities of pure ethyl alcohol are given by Ramsay and Young. [53]

Densities of sugar solutions of Table 14 have been calculated from the data of Plato, [54] extrapolation being necessary for temperatures above 60° C (140° F). The change in density for a difference in temperature of 5° C (9° F) was found to be within 0.0004 of the difference in density calculated from the data of Gerlach. [55]

[49] Circular No. 19, Bureau of Standards; pp. 42, 43, 5th ed.; 1916.
[50] N. S. Osborne, Scientific Paper No. 197, Bureau of Standards, pp. 424, 402; 1913.
[51] Physikalisch-chemische Tabellen, p. 303; 1912.
[52] F. Schwers, Recueil des travaux chimiques des Pays-Bas, 28, p. 263; 1909.
[53] W. Ramsay and S. Young, Phil. Trans. R. S., 177, pt. 1, p. 136, 1886; S. Young, Sci. Proc. Royal Dublin Soc., 12, p. 441, 1909-10.
[54] F. Plato, Zeit. des Vereines der deutschen Zucker-Industrie 37, p. 1079, 1900; Landolt-Börnstein, loc. cit., p. 311; E. O. von Lippmann, Die Chemie der Zuckerarten, 2, p. 1078, 1904.
[55] Th. Gerlach, Zeit. des Vereines fur die Rubenzucker-Industrie, 14, p. 354, 1864; Dingler's Polytechnisches Journal, 172, p. 31, 1864.

TABLE 13

Ethyl Alcohol Solutions

TEMPERATURE, 0° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	55.8	1.792	0.99987	1.792	0.99985
10	30.2	3.311	.98479	3.362	.98472
20	18.8	5.319	.97551	5.453	.97553
30	14.4	6.94	.96524	7.195	.96488
39	13.8	7.25	.95113	7.619	.95092
40	14.0	7.14	.94930	7.524	.94916
45	14.4	6.94	.93954	7.391	.93962
50	15.2	6.58	.92917	7.080	.92928
60	17.4	5.75	.90720	6.335	.90715
70	21.0	4.762	.88420	5.386	.88427
80	27.1	3.690	.86030	4.289	.86044
90	36.6	2.732	.83498	3.272	.83506
100	56.4	1.773	.80627	2.199	.80597

TEMPERATURE, 5° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	65.8	1.519	0.99999	1.519	....
10	38.8	2.577	.98452	2.618	....
20	24.6	4.065	.97413	4.173	....
30	18.9	5.29	.96257	5.497	....
39	17.8	5.62	.94776	5.928	....
40	17.9	5.59	.94588	5.906	....
45	18.2	5.50	.93593	5.871	....
50	19.0	5.26	.92542	5.687	....
60	21.6	4.63	.90326	5.125	....
70	25.6	3.906	.88013	4.438	....
80	32.0	3.125	.85615	3.650	....
90	43.3	2.309	.83078	2.780	....
100	61.6	1.623	.80206	2.024	....

TEMPERATURE, 10° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	76.5	1.308	0.99973	1.308	....
10	45.9	2.179	.98393	2.214	....
20	31.6	3.165	.97252	3.254	....
30	24.7	4.05	.95977	4.218	....
39	22.8	4.39	.94431	4.645	....
40	22.8	4.39	.94238	4.654	....
45	23.0	4.35	.93226	4.664	....
50	23.9	4.18	.92162	4.540	....
60	26.5	3.77	.89927	4.196	....
70	30.6	3.268	.87602	3.730	....
80	36.9	2.710	.85197	3.181	....
90	47.6	2.101	.82654	2.542	....
100	68.2	1.466	.79784	1.838	....

TEMPERATURE, 15° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	87.7	1.140	0.99913	1.141	....
10	55.8	1.792	.98304	1.823	....
20	38.2	2.618	.97068	2.697	....
30	30.7	3.26	.95686	3.404	....
39	28.4	3.52	.94079	3.743	....
40	28.3	3.53	.93882	3.764	....
45	28.5	3.51	.92852	3.779	....
50	29.1	3.44	.91776	3.744	....
60	31.8	3.14	.89523	3.513	....
70	36.1	2.770	.87187	3.177	....
80	43.3	2.309	.84772	2.724	....
90	55.5	1.802	.82227	2.191	....
100	75.1	1.332	.79360	1.678	....

TABLE 13—Continued

Ethyl Alcohol Solutions—Continued

TEMPERATURE, 20° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	99.5	1.005	0.99823	1.007	....
10	65.0	1.538	.98187	1.567	....
20	45.8	2.183	.96864	2.254	....
30	36.9	2.71	.95382	2.841	....
39	34.7	2.88	.93720	3.075	....
40	34.4	2.91	.93518	3.108	....
45	34.7	2.88	.92472	3.116	....
50	34.8	2.87	.91384	3.144	....
60	37.4	2.67	.89113	3.000	....
70	42.2	2.370	.86766	2.731	....
80	49.8	2.008	.84344	2.381	....
90	62.1	1.610	.81797	1.969	....
100	83.3	1.200	.78934	1.521	....

TEMPERATURE, 25° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	111.9	0.894	0.99708	0.896	....
10	75.6	1.323	.98043	1.349	....
20	55.1	1.815	.96639	1.878	....
30	45.9	2.18	.95067	2.292	....
39	42.5	2.35	.93353	2.520	....
40	42.5	2.35	.93148	2.526	....
45	41.9	2.39	.92085	2.592	....
50	41.7	2.40	.90985	2.636	....
60	44.6	2.24	.88699	2.528	....
70	49.1	2.037	.86340	2.359	....
80	57.2	1.748	.83911	2.083	....
90	70.2	1.424	.81362	1.751	....
100	91.2	1.096	.78506	1.397	....

TEMPERATURE, 30° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	124.9	0.801	0.99568	0.804	....
10	86.2	1.160	.97875	1.185	....
20	64.4	1.553	.96395	1.611	....
30	53.4	1.87	.94741	1.977	....
39	50.0	2.00	.92979	2.151	....
40	49.4	2.02	.72770	2.182	....
45	49.5	2.02	.91692	2.203	....
50	49.6	2.02	.90580	2.226	....
60	51.9	1.93	.88278	2.183	....
70	56.6	1.767	.85908	2.057	....
80	65.3	1.531	.83473	1.835	....
90	78.2	1.279	.80922	1.580	....
100	99.7	1.003	.78075	1.285	....

TEMPERATURE, 35° C

Per cent alcohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt–Börnstein
0	138.4	0.723	0.99406	0.727	....
10	99.4	1.006	.97685	1.030	....
20	75.1	1.332	.96134	1.385	....
30	63.3	1.58	.94403	1.673	....
39	58.6	1.71	.92597	1.843	....
40	58.3	1.72	.92385	1.857	....
45	57.7	1.73	.91291	1.898	....
50	58.0	1.72	.90168	1.912	....
60	60.1	1.66	.87851	1.894	....
70	65.4	1.529	.85470	1.789	....
80	73.8	1.355	.83029	1.632	....
90	87.2	1.147	.80478	1.425	....
100	109.4	.914	.77641	1.177	....

TABLE 13—Continued

Ethyl Alcohol Solutions—Continued

TEMPERATURE, 40° C

Per cent alchohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt-Börnstein
0	152.5	0.656	0.99225	0.661	....
10	110.2	.907	.97475	.931	....
20	86.2	1.160	.95856	1.210	....
30	73.1	1.368	.94055	1.454	....
39	67.9	1.473	.92208	1.597	....
40	67.5	1.482	.91992	1.610	....
45	66.9	1.495	.90884	1.645	....
50	66.7	1.499	.89750	1.670	....
60	69.1	1.447	.87417	1.655	....
70	74.4	1.344	.85025	1.581	....
80	83.1	1.203	.82578	1.457	....
90	96.6	1.035	.80028	1.294	....
100	119.9	.834	.77203	1.080	....

TEMPERATURE, 45° C

Per cent alchohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt-Börnstein
0	167.0	0.599	0.99024	0.605	....
10	123.2	.812	.97247	.835	....
20	98.5	1.015	.95561	1.062	....
30	84.1	1.189	.93696	1.269	....
39	77.9	1.284	.91810	1.398	....
40	77.6	1.289	.91592	1.407	....
45	76.5	1.307	.90471	1.445	....
50	77.3	1.294	.89325	1.448	....
60	78.7	1.271	.86976	1.461	....
70	84.1	1.189	.84573	1.406	....
80	92.5	1.081	.82121	1.316	....
90	106.5	.939	.79572	1.180	....
100	130.8	.764	.76761	.996	....

TEMPERATURE, 50° C

Per cent alchohol by weight	Fluidity	Viscosity in cp	Density from B. S. data	100× kinematic viscosity	Density from data of Landolt-Börnstein
0	182.0	0.549	0.98807	0.556	....
10	136.3	.734	.97002	.756	....
20	110.2	.907	.95253	.953	....
30	95.2	1.050	.93327	1.126	....
39	89.0	1.124	.91406	1.229	....
40	88.3	1.132	.91185	1.242	....
45	87.1	1.148	.90050	1.275	....
50	86.6	1.155	.88893	1.299	....
60	88.7	1.127	.86527	1.303	....
70	94.2	1.062	.84114	1.262	....
80	103.3	.968	.81657	1.186	....
90	117.9	.848	.79110	1.072	....
100	142.5	.702	.76315	.920	....

TABLE 14

Sucrose Solution

Temperature, °C	Per cent sucrose by weight	Fluidity	Viscosity in cp	Density	100× kinematic viscosity
0........	0	55. 80	1. 7921	0. 99987	1. 7923
	20	26. 29	3. 804	1. 08546	3. 504
	40	6. 77	14. 77	1. 18349	12. 48
	60	. 42	238	1. 29560	184
5........	0	65. 84	1. 5188	0. 99999	1. 5188
	20	31. 71	3. 154	1. 08460	2. 908
	40	8. 65	11. 56	1. 18192	9. 78
	60	. 64	156	1. 29341	121
10........	0	76. 47	1. 3077	0. 99973	1. 3081
	20	37. 71	2. 652	1. 08353	2. 447
	40	10. 21	9. 794	1. 18020	8. 299
	60	. 91	109. 8	1. 29117	85. 11
15........	0	87. 69	1. 1404	0. 99913	1. 1414
	20	44. 11	2. 267	1. 08233	2. 095
	40	13. 39	7. 468	1. 17837	6. 338
	60	1. 34	74. 6	1. 28884	57. 90
20........	0	99. 50	1. 0050	0. 99823	1. 0068
	20	51. 02	1. 960	1. 08094	1. 813
	40	16. 13	6. 200	1. 17648	5. 270
	60	1. 77	56. 5	1. 28644	43. 92
25........	0	111. 91	0. 8937	0. 99708	0. 8962
	20	58. 69	1. 704	1. 07940	1. 579
	40	19. 28	5. 187	1. 17439	4. 417
	60	2. 28	43. 86	1. 28399	34. 16
30........	0	124. 89	0. 8007	0. 99568	0. 8043
	20	66. 51	1. 504	1. 07767	1. 395
	40	22. 82	4. 382	1. 17214	3. 739
	60	2. 96	33. 78	1. 28144	26. 36
35........	0	138. 40	0. 7225	0. 99406	0. 7269
	20	75. 12	1. 331	1. 07575	1. 237
	40	26. 58	3. 762	1. 16994	3. 216
	60	3. 77	26. 52	1. 27882	20. 74
40........	0	152. 45	0. 6560	0. 99225	0. 6611
	20	83. 82	1. 193	1. 07366	1. 111
	40	30. 78	3. 249	1. 16759	2. 783
	60	4. 70	21. 28	1. 27615	16. 67
45........	0	167. 00	0. 5988	0. 99024	0. 6047
	20	93. 42	1. 070	1. 07141	. 9991
	40	35. 13	2. 847	1. 16508	2. 443
	60	5. 82	17. 18	1. 27341	13. 49
50........	0	182. 00	0. 5494	0. 98807	0. 5561
	20	103. 07	. 970	1. 06898	. 9076
	40	40. 05	2. 497	1. 16248	2. 148
	60	7. 14	14. 01	1. 27058	11. 02